International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

IJIEMR Transactions, online available on 8th Feb 2021. Link :http://www.ijiemr.org/downloads.php?vol=Volume-10&issue=ISSUE-02

DOI: 10.48047/IJIEMR/V10/I02/07

Title NUMERICAL RECOVERY OF A FUNCTION FROM THE VALUES OF ITS INTEGRALS ON A FAMILY OF SEGMENTS

Volume 10, Issue 02, Pages: 29-32.

Paper Authors

G. Djaykov, A. Bekbosinov, H. Esbergenov

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

NUMERICAL RECOVERY OF A FUNCTION FROM THE VALUES OF ITS INTEGRALS ON A FAMILY OF SEGMENTS

G. Djaykov, A. Bekbosinov, H. Esbergenov
Nukus branch of the Tashkent University of Information Technologies

Abstract. We study the problems of integral geometry in a strip on a family of line segments with a given weight function. In the first section, an analytic representation of the solution in the class of smooth compactly supported functions is obtained. Estimates for the solution of the problem in Sobolev spaces are presented, which implies its weak ill-posedness. Further, the obtained theoretical results are investigated by experimental data.

1. INTRODUCTION

In this paper, we study problems of integral geometry in a strip on a family of line segments with a given weight function. In the first part, analytical representation of the solution is obtained in the class of smooth compactly supported functions. Estimate for the solution of the problem in spaces is presented, which implies its weak ill-posedness.

In the second part, obtained theoretical results are investigated by experimental data using various software packages. An approximate solution to the problem is sought in a uniform grid in the region $D=[a,b]\times[c,d]$. The solution method is based on the use of finite-difference methods for numerical differentiation in partial derivatives. We construct an algorithm to solve the set problem. Numerical and graphical results of applying these algorithms to solving the problem are presented.

The problem of recovering a function from known integrals of it along all possible hyperplanes in Euclidean space was considered in [1].

A uniqueness theorem and a stability estimate are obtained for the problem with perturbation in [2,3].

2. INVERSION FORMULA

Let us denote $\Omega=\{(x,y): x\in R^1, 0\le y\le H\}$.

For everyone (x,y) lying in the strip Ω.

$$\Gamma(x,y)=\{(\xi,\eta): x-\xi=y-\eta,\ 0\le\eta\le y\le H\}$$

Consider the operator equation

$$\int\limits_{\Gamma(x,y)} g(x,y,\xi,\eta)u(\xi,\eta)ds=f(x,y), \tag{1}$$

Statement of the problem. Reconstruct a function of two variables $u(x,y)$ from equations (1), if the integrals of it with a given weight function $g(x,\xi)=x-\xi$ on a family of curves are known.

Let us write equation (1) in the following form:

$$\int\limits_0^y u(x-h,\eta)(y-\eta)d\eta=f(x,y), \tag{2}$$

Let us apply the Fourier transform concerning the variable x to both sides of equation (2):

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

$$\hat{f}(\lambda,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{i\lambda x}\int_0^y u(x-h,\eta)(y-\eta)\,d\eta dx$$
,

$$\int_0^y \hat{u}(\lambda,\eta)(y-\eta)e^{i\lambda(y-\eta)}d\eta=\hat{f}(\lambda,y).$$

(3)

We apply the Laplace transform for the variable y to the last equation:

$$\tilde{\hat{f}}(\lambda,p)=\int_0^{+\infty}e^{-py}\int_0^y \hat{u}(\lambda,\eta)(y-\eta)e^{i\lambda(y-\eta)}d\eta dy=$$

$$=\int_0^{+\infty}\tau\cdot e^{-(p-i\lambda)\tau}d\tau\cdot\int_0^{+\infty}\hat{u}(\lambda,\eta)e^{-p\eta}d\eta=I(\lambda,p)\cdot\tilde{\hat{u}}(\lambda,p)$$

Here $\tilde{\hat{u}}(\lambda,p)$ and $\tilde{\hat{f}}(\lambda,p)$ the Laplace transform of the function $\hat{u}(\lambda,y),\hat{f}(\lambda,y)$ respectively.

$$I(\lambda,p)=\int_0^{+\infty}\tau\cdot e^{-(p-i\lambda)\tau}d\tau=\frac{1}{(p-i\lambda)^2},\ \operatorname{Re}p>0.$$

$$\tilde{\hat{u}}(\lambda,p)=\frac{1}{I(\lambda,p)}\tilde{\hat{f}}(\lambda,p),$$

$$\tilde{\hat{u}}(\lambda,p)=(p-i\lambda)^2\,\tilde{\hat{f}}(\lambda,p).$$

or

$$\tilde{\hat{u}}(\lambda,p)=\left(p^2-2pi\lambda-\lambda^2\right)\tilde{\hat{f}}(\lambda,p).$$

(4)

Applying to (4) the inverse Laplace and Fourier transform in the variable p and λ, we get:

$$u(x,y)=\frac{\partial^2}{\partial x^2}f(x,y)+2\frac{\partial^2}{\partial x\partial y}f(x,y)+\frac{\partial^2}{\partial y^2}f(x,y)$$

(5)

Hence follows the following estimate

$$\|u(x,y)\|_{L_2}\le C_1\|f(x,y)\|_{W_2^{2,2}}$$

where C_1 – are some constant.

3. NUMERICAL EXPERIMENT

The obtained theoretical results in the first paragraph are investigated by experimental studies using the Fortran program.

Let's introduce a uniform mesh in a rectangular area $D=[a,b]\times[c,d]$. We find for approximate solutions to the problem on this rectangle. The solution method is based on the application of the method of finite-difference schemes for numerical differentiation in partial derivatives. Let's divide the area D into n_x,n_y parts corresponding to the axes x, y.

In general, problems of numerical differentiation have two aspects. First, this operation has to be performed in cases where information about the function under study is given in tabular form or as a set of experimental research results. Second, a kind of numerical differentiation is carried out when constructing a discrete analog of ordinary differential equations or partial differential equations, which cannot be solved analytically due to the nonlinearity or complex geometry of the investigated area. This section deals mainly with the details of numerical differentiation by given tabular data.

Example. In particular, D the area can be written in the following way, if the extreme points of the segments $[a,b]$ and $[c,d]$ are given:

$$D=\{-1\le x\le 1,\ 0\le y\le 2,\ x_i=-1+ih_x,\ y_j=jh_j\}$$

We will restrict ourselves to considering two typical examples. As a test example for comparison with the results of numerical calculations, we use the following functions

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

$$u_1(x,y)=\left(x^2-1\right)\left(y^2-2y\right),$$

The functions $f(x,y)$ are obtained from formula (2). For an approximate value of second-order differentiation, we can take the following difference relations:

$$\frac{\partial^2}{\partial x^2}f(x,y)\approx\frac{f_{i+1j}-2f_{ij}+f_{i-1j}}{h_x^{\,2}},\quad \frac{\partial^2}{\partial y^2}f(x,y)\approx\frac{f_{ij+1}-2f_{ij}+f_{ij-1}}{h_y^{\,2}},$$

$$\frac{\partial}{\partial x}f(x,y)\approx\frac{f_{i+1j}-f_{i-1j}}{2h_x},$$

$$\frac{\partial}{\partial y}f(x,y)\approx\frac{f_{ij+1}-f_{ij-1}}{2h_y}$$

Equations (5) with the second-order of accuracy can be represented in the finite-difference form:

$$u_{ij}^{\Pi}=\frac{f_{ij+1}-2f_{ij}+f_{ij-1}}{h_y^{\,2}}+\frac{f_{i+1j+1}-f_{i-1j+1}}{2h_xh_y}-\frac{f_{i+1j-1}-f_{i-1j-1}}{2h_xh_y}+\frac{f_{i+1j}-2f_{ij}+f_{i-1j}}{h_x^{\,2}}. \quad (6)$$

When calculating derivatives of a higher order, when the denominator of the difference ratio is included h^k, $k>1, u_{ij}$ the influence of inaccuracy in the task is stronger. This subtlety is associated with the incorrectness of the differentiation problem.

The scheme of the algorithm for solving the problem is as follows:

Step 1. Divide the segment $[a,b]$ on the axis Ox and $[c,d]$ Oy the axis into n_x and n_y parts, $x_i=a+ih_x$, $y_j=b+jh_y$ respectively.

Step 2. We will denote u_{ij}^{Π} the approximations of functions $u(x_i,y_j)$.

$$u_{ij}^{\Pi}=\frac{f_{ij+1}-2f_{ij}+f_{ij-1}}{h_y^{\,2}}+\frac{f_{i+1j+1}-f_{i-1j+1}}{2h_xh_y}-\frac{f_{i+1j-1}-f_{i-1j-1}}{2h_xh_y}+\frac{f_{i+1j}-2f_{ij}+f_{i-1j}}{h_x^{\,2}}$$

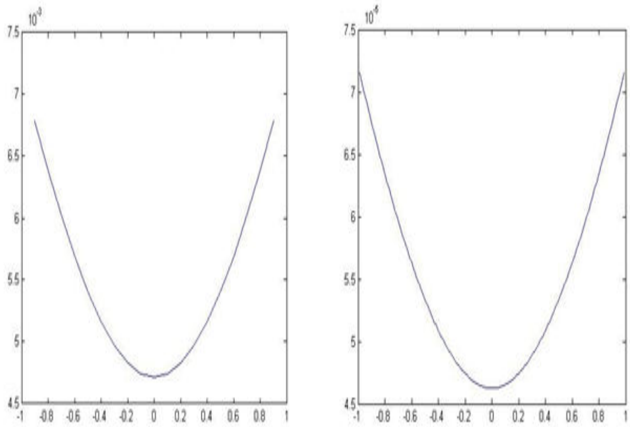
Step 3. Let's calculate the error value δ_1 – as follows:

Let's designate: i - row number, j - column number.

$$(\delta_1)_i=\sqrt{\frac{1}{nx-1}\sum_{j=1}^{ny-1}\left|u_{ij}^{T}-u_{ij}^{\Pi}\right|^2}\quad i=\overline{1,n_x-1}, \quad (7)$$

where u_{ij}^{T} – is the exact solution, u_{ij}^{Π} – the approximate solution.

To illustrate the calculations, as well as the accuracy achieved, we apply this process in Figure 1 with steps $h=0.1$ and $h=0.01$. The calculations show (Figure 1) that the error values $(\delta_1)_i$ -at the beginning and the end of the nodes on the segment are greater than the values of the nodes near the centers of the segment.



1-fig.

change in error $(\delta_1)_i$ with steps $h_x=h_y=0.1$, $h_x=h_y=0.01$

CONCLUSION

We considered problems of integral geometry in a strip on a family of line segments with a given weight function. An analytical representation of the solution is obtained in the class of smooth compactly supported functions. We construct an algorithm to solve the set problem.

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

Based on the approximate values obtained, we can conclude that the calculations expect two correct digits after the decimal point at $h_x = h_y = 0.1$, four zeros after the decimal point at $h_x = h_y = 0.01$. The error graph shows that the values of the derivatives at the nodes located at the beginning and at the end of the line segment are calculated with less accuracy than the values of the derivatives at the nodes located near the center of the line segment. Thus, summarizing the above, it can be noted that the method of formal replacement of derivatives by finite-difference relations is simple and stereotyped. The difference analogue of the differential equation obtained with its help approximates this equation at the grid point.

Литература

1. J. Radon *Über die BestimmungvorFunktionendurchihreIntegralwärtelängsgewisserMäannigfritigkeiten*, Ber. Verb. Sächs. Akad. (1917), 262–277.
2. A. Begmatov, G. Djaykov, *Linear problem of integral geometry in the strip with smooth weight functions and perturbation*, Vladikavkaz. Mat. J. (2015), 14–22.
3. A. Begmatov, G. Djaykov, *Numerical recovery of function in a strip from given integral data on linear manifolds*, Proceedings of the International forum on strategic technology, Novosibirsk, June 2016, 478-483.