The Inverse of the Scattering Theory for the System of Direct Equations on the Whole Number

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Annotation
The inverse of the scattering theory for the Dirac operator on the integer axis is considered under the following conditions:
a) tends to zero fast enough in real continuity and functions;
b) The spectrum of the Dirac operator on the axis is only discrete.
The thesis provides information about the scattering function of the coefficients in the interval, and the recovery algorithm in the interval on the spectral function.

Keywords: Dirac operator, Weil – Titchmarsh function, spectral function, Yost solution, Gelfand – Levitan algorithm, potential matrix function.

Suppose that the inverse of the scattering theory is given for the following Dirac operator on the whole number axis.

$$Dy \equiv B\frac{dy}{dx} + \Omega(x)y = \lambda y, \qquad -\infty < x < \infty \qquad (1)$$

here

$$y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}.$$

$p(x)$ and $q(x)$ real continuous functions that satisfy the following conditions:

(a) in arbitrary real limits

$$|p(x)| \le \frac{C}{(1+|x|)^{2+\varepsilon}}, \qquad |q(x)| \le \frac{C}{(1+|x|)^{1+\varepsilon}}, \qquad x \in [a, \infty),$$

$C > 0$ and $\varepsilon > 0$ are constant numbers;

(b) The spectrum of the following boundary value problems is discrete only:

$$\begin{cases} Dy = \lambda y, & -\infty < x < a, \\ y_1(a) = 0. \end{cases} \qquad (2)$$

Consider the following initial conditions of equation (1)

$$\theta(a,\lambda) = (1,\ 0)^{T}, \qquad \varphi(a,\lambda) = (0,\ -1)^{T}$$

satisfactory solutions $\theta(x,\lambda)$ and $\varphi(x,\lambda)$ let. Then the Weil solution of problem (2) is a

$\tilde{\psi}(x,\lambda)$ value with the following condition

$$\tilde{\psi}(x,\lambda) = \theta(x,\lambda) + m_{-}(\lambda)\varphi(x,\lambda) \in L_2^2(-\infty, a)$$

where (2) does not belong to the spectrum of the solution. The function (2) is called the $m_{-}(\lambda)$ Weil – Titchmarsh function of the problem.

(1) a solution of the system that satisfies the following condition

$$f(x,\lambda) = \begin{pmatrix} i \\ 1 \end{pmatrix} e^{i\lambda x} + \bar{o}(1), \quad x \to +\infty \quad (\operatorname{Im} \lambda = 0),$$

It is called the Yost solution.

Theorem 1. If condition (a) is valid, then the Yost solution exists and is unique, and it is in the form of the following integral expression

$$f(x,\lambda) = \begin{pmatrix} i \\ 1 \end{pmatrix} e^{i\lambda x} + \int_x^\infty A(x,t) \begin{pmatrix} i \\ 1 \end{pmatrix} e^{i\lambda t} dt \qquad (3)$$

where it does not $A(x,t) = \begin{pmatrix} A_{11}(x,t) & A_{12}(x,t) \\ A_{21}(x,t) & A_{22}(x,t) \end{pmatrix}$ depend on the nucleus and satisfies the following conditions:

1) $B A'_x(x,t) + A'_t(x,t) B = -\Omega(x) A(x,t)$,

2) $B A(x,x) - A(x,x) B = \Omega(x)$,

3) $\left| A_{12}(x,t) \right| + \left| A_{21}(x,t) \right| \le \dfrac{C_1}{\left(1+|x|\right)\left(1+|t|\right)^{1+\varepsilon}}$,

$$\left| A_{11}(x,t) \right| + \left| A_{22}(x,t) \right| \le \frac{C_1}{\left(1+|t|\right)^{1+\varepsilon}},,$$

here $C_1 > 0$ and $\varepsilon > 0$ some do not change.

Let $\lambda \in R^1$ and (a), (b) be reasonable. Then

$$u(x,\lambda) = S(\lambda) f(x,\lambda) - \overline{f(x,\lambda)}, \quad , (4)$$

here

$$u(x,\lambda) = \frac{2i}{W(\lambda)} \psi(x,\lambda), \qquad S(\lambda) = \frac{\overline{W(\lambda)}}{W(\lambda)}, \qquad W(\lambda) = W\{\psi(x,\lambda), f(x,\lambda)\},$$

and vronskiani of vector-functions, (1) is called the scattering function of the problem.

We will introduce the function in the following form

$$S_0(\lambda) = e^{-2ia\lambda} \cdot \frac{\psi_1(a,\lambda) + i\psi_2(a,\lambda)}{\psi_1(a,\lambda) - i\psi_2(a,\lambda)}. \quad (5)$$

With the method of V.A. Marchenko (3) gives a linear integral equation for the core of the expression

$$f(x,\lambda) = \begin{pmatrix} i \\ 1 \end{pmatrix} e^{i\lambda x} + \int_x^{\infty} A(x,t) \begin{pmatrix} i \\ 1 \end{pmatrix} e^{i\lambda t} dt$$

Theorem 2. For each assignment, the function satisfies the Gelfand-Levitan-Marchenko integral equation

$$\begin{pmatrix} i \\ 1 \end{pmatrix} G(x+y) + \int_x^{\infty} A(x,t) \begin{pmatrix} i \\ 1 \end{pmatrix} G(t+y) dt - A(x,y) \begin{pmatrix} -i \\ 1 \end{pmatrix} = 0, \quad (x < y < \infty),$$

here

$$G(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [S(\lambda) - S_0(\lambda)] e^{i\lambda t} d\lambda .$$

Using the formula for the relationship between the Weil-Titchmarsh function and the spectral function given in the conclusion (2), the problem can be reconstructed by the Gelfand-Levitan algorithm [1,2].

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