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Phase Space Reconstruction of EEG Time Series using Epilepsy Data

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Abstract:

Time series of a system replicates its behavior. Studies on time series help in analyzing the nature of the dynamical systems. Among all the methods used to analyze time series, phase space reconstruction is one, where its trajectory in phase space helps in looking into the system. In physics, the phase space of a dynamical system depicts the system evolution. A straightforward way to figure out the dynamics of a system is, by solving its equations of motion. Time series evolving from complex systems like heart, brain, sunspots etc., are challenging. Assuming the time series of some variable generated by a dynamical system, one can reconstruct the phase space. Delay coordinate embedding is one of the methods used to reconstruct the phase space trajectory of time series. Phase space of a time series allows analyzing the nonlinearity of time series by measuring the invariant properties like the Lyapunov exponents, entropies, correlation and fractal dimensions. We discussed the methods in order to reconstruct the phase space trajectory of the univariate EEG time series.

Keywords: EEG signals, embedded dimension, embedded time-delay, phase-space, reconstruction, time series.

1. Introduction

The analysis of signals generated from complex systems is an intricate process and choosing methods to uncover the underlying behavior of systems is challenging. Time series of systems like heart, brain and sunspots are widely studied using the methods of nonlinear analysis [1]. Due to the complexity of time series, the concepts of chaos theory translates its underpinned nature [2]. Using of chaos theory has an increasing frequency in the area of health sciences [3]. For any chaotic system, it is possible to find the deterministic nature of a system by investigating its time series. The brain, one such complex system, whose time series recorded by electroencephalogram (EEG) are nonlinear. The nonlinear analysis of the brain signals is important in understanding its neurophysiology [4]. EEG has been used in various clinical applications in identifying sleep stages and to classify the normal and abnormal brains in patients with Alzheimer's Disease (AD) [5]. The chaotic analysis helps in studying the epileptic recordings of ictal, pre-ictal and post-ictal segments, using the nonlinear parameters like the Lyapunov exponents, entropies, correlation and fractal dimensions [6 - 9]. In non-linear methods, phase space reconstruction has been used to embed the time series in phase space for further analysis [10 - 12]. Phase space of a time series requires calculating the parameters, embedding time delay (τ) and embedding dimension(m) [13 - 15]. Various methods are available to calculate these parameters to fit the time series in phase space. Phase space reconstruction of biomedical signals significantly classifies the variability of signals generated from different states of health conditions, behavioral states etc. In this paper, we discussed the phase space reconstruction methods and applied them to draw the phase space plots of EEG signals.

2. Electroencephalogram (EEG)

EEG is a non-invasive neuro imaging modality which measures the electrical activity of brain. It' high temporal resolution captures the significant changes in dynamical brain. Cerebral cortex, an outer surface of brain, have complex functionality like, thought-process, action etc. Variations in EEG recorded signals in subjects, reliably helps in classifying the states of pathological functioning in comparison with normals. Epilepsy is the most common neurological disorder which arises due to non-functioning of brain regions, due to brain tumors, strokes etc. EEG has been widely used to study the episodes of epileptic subjects based on pre-ictal, ictal and post-ictal intervals [16]. Onset recording of epileptic nature helps in pre-surgical medication in subjects. EEG predicts epileptic behavior in autism patients [17]. The measured signals from EEG reveals the chaotic nature of the brain and such chaotic systems are well analyzed through nonlinear studies.

3. Time series analysis

3.1. Phase Space Reconstruction

Any system's properties are represents by variables, which can have a range of values. Each value possessed by the system represents the state of the system. The time evolution of states depicts a trajectory in the coordinate space called phase space. A system is dynamical, when the state of it evolves with time, by obeying some rule. By giving some initial conditions to it, the evolution with time generates a trajectory in the phase space. Chaos theory helps in determining the dynamics of systems. A dynamical system is chaotic, when the state of the system used as a new initial condition. By using the context of embedding theorem [18, 19] enables us to reconstruct the phase space of a univariate time series. According to embedding theorem, each point in the time series represents state of the system and is written as scalar sequence

$$x(t) = \{x_0, x_1, x_2, x_3, \dots \dots \dots, x_i, \dots . x_N\}, \quad (1)$$

where x_i is the measured value of the variable at time t. Takens theorem [18] suggested that, each point in the time series represents a vector in the reconstructed space

$$y_i(m) = \left\{x_i, x_{i+1}, x_{i+2}, \dots \dots, x_{i+(m-1)\tau}\right\}, \quad (2)$$

where $y_i(m)$ is the i^{th} reconstructed vector and τ, m are embedded delay and embedded dimension respectively. According to the Takens theorem, suitably large value of m, helps in one-to-one mapping of the attractor and original system, which is the evidence for invariance of nonlinear properties of measured signal in phase space.

The successful estimation of τ and m, is required in reconstruction of attractor. Choosing τ value follows the criteria that, at first its value is large enough so that the delay between x_i and

x_{i+1} differ from each other and it provides the information between the interaction of components of the system. In second case, τ should not be larger than typical values so that it avoids losing information about its initial state. Similarly, m suggests the dimension of phase space to fit the reconstructed attractor [20].

Time delay constant:

There is no restriction on choosing the value of time delay constant τ [21]. Fraser and Swinney [22] suggested that using mutual information between x_i and x_{i+1}, one could find the amount of information about x_{i+1} by presuming we know about x_i.

$$I(\tau) = \sum_{n=1}^{N} P_{AB}(x_i, x_{i+\tau}) \log_2 \left[\frac{P_{AB}(x_i, x_{i+\tau})}{P_A(x_i)P_B(x_{i+\tau})}\right], \quad (3)$$

where $P_A(x_i)$ and $P_B(x_{i+\tau})$ are probabilities of occurrence of variable x_i and $x_{i+\tau}$ in systems A and B and $P_{AB}(x_i, x_{i+\tau})$ is joint probability of x_i and $x_{i+\tau}$ in A, B. As τ increases, $I(\tau)$ decreases and then it usually rises again. At this moment, Fraser and Swinney suggested the first minimum of $I(\tau)$, to select the value of τ.

3.2. Embedded Dimension

A minimum number of dimension is required to fit the given scalar time series. Methods like, finding the invariants on attractor while increasing the dimension [23], singular value decomposition (SVD) [24] and the method of false nearest neighbors (FNN) [25], are independent of delay constant τ. In comparison with these methods, Caos [26] proposed that a good choice of embedding dimension is in contrast with embedded time delay. Remarkably, different choice of time delay constants leads to different minimum dimensions, a good choice of time delay constant (τ) necessary. Caos used equation (4) to determine the value of embedded dimension, m.

$$E(m) = \frac{1}{N - m\tau} \sum_{i=1}^{N-m\tau} a(i, m), \quad (4)$$

where (i, m) is the ratio of maximum norm (Euclidean distance) of i^{th} and $n(i, m)^{th}$ reconstructed vector in m and m+1 embedding dimension

$$a(i, m) = \left\| \frac{y_i(m+1) - y_{n(i,m)}(m+1)}{y_i(m) - y_{n(i,m)}(m)} \right\| \quad (5)$$

and $n(i, m)$ $(1 \leq n(i, m) \leq N - m\tau)$ is an integer such that $y_{n(i,m)}(m)$ is the nearest neighbour (NN) of $y_i(m)$. The invariance of distance between any two neighbouring points in phase space are analyzed using the ratio

$$E1(m) = \frac{E(m+1)}{E(m)} \quad (6)$$

There is no change in $E1(m)$ when m is greater than some value m_0. Then (m+1) is the minimum embedding dimension to fit the scalar time series.

4. Results and discussion

Firstly, we applied the time series methods to the well-known system, a simple sinusoidal function to check its validity. Sinusoidal functions are continuous functions whose phase space trajectories are periodic and limits to 2D space. Mutual information and Caos functions are confirmed periodic nature of sinusoidal waves, whose trajectory is in a plane. Figure 1 shows the simple phase space reconstruction of a sine wave. In figure 1, phase space reconstruction of a periodic signal is plotted with

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the embedding delay and embedding dimension. The same functions applied to an EEG recorded signal, whose attractor confirms the chaotic nature of brain electrical activity. Onset activity of epileptic signals can be well studied under nonlinear time series analysis, because EEG demonstrates the physiological aspects of abnormalities causes in cortical activity. EEG has also been used to classify the seizure disorders like idiopathic, focal etc.

Figure 1 and 2 shows the reconstructed attractor for aperiodic and EEG signal, where a) represents the time series of signal, b) is measured embedded time delay value, c) is embedded dimension from Caos method and d) is reconstructed attractor of time series respectively.

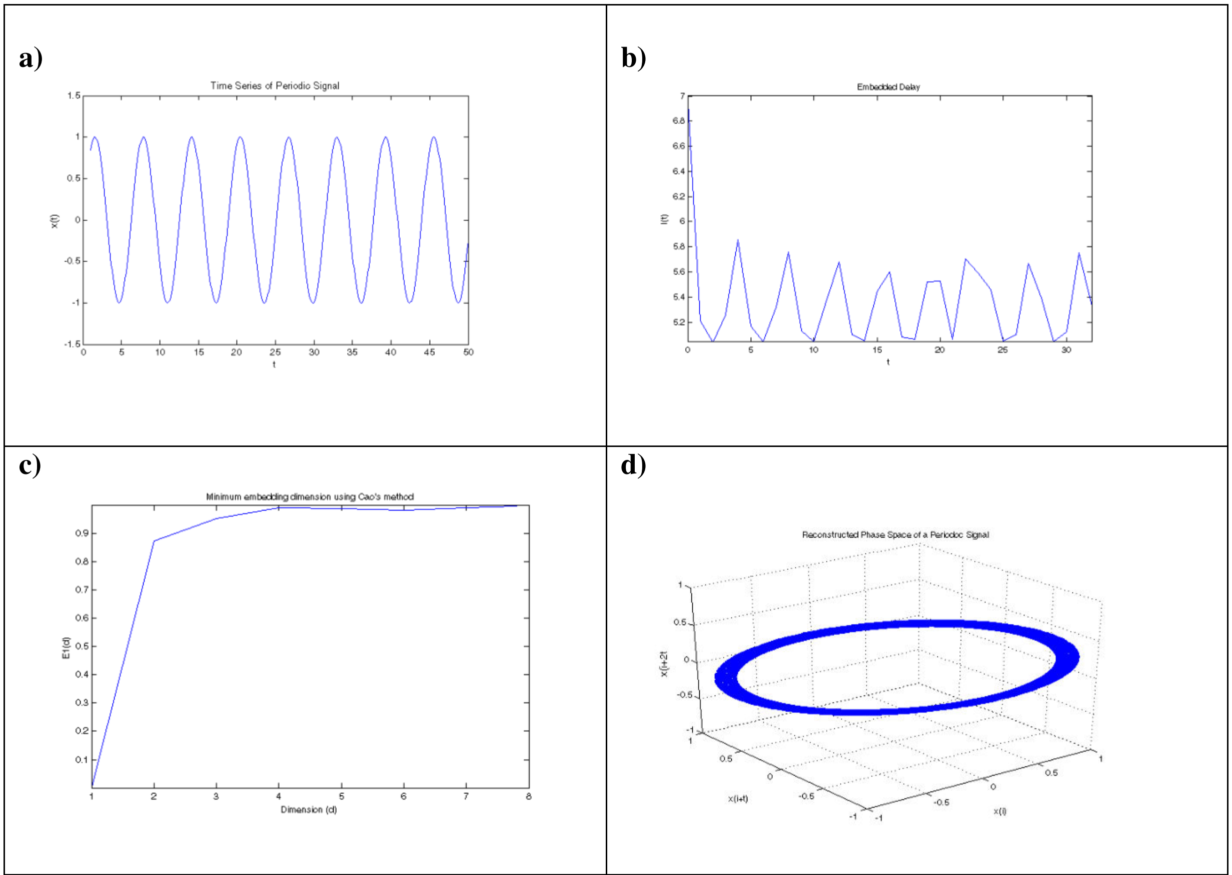

Figure: 1. (a) time series of sinusoidal signal, (b) embedded delay, (c) embedded dimension, (d) phase space plot

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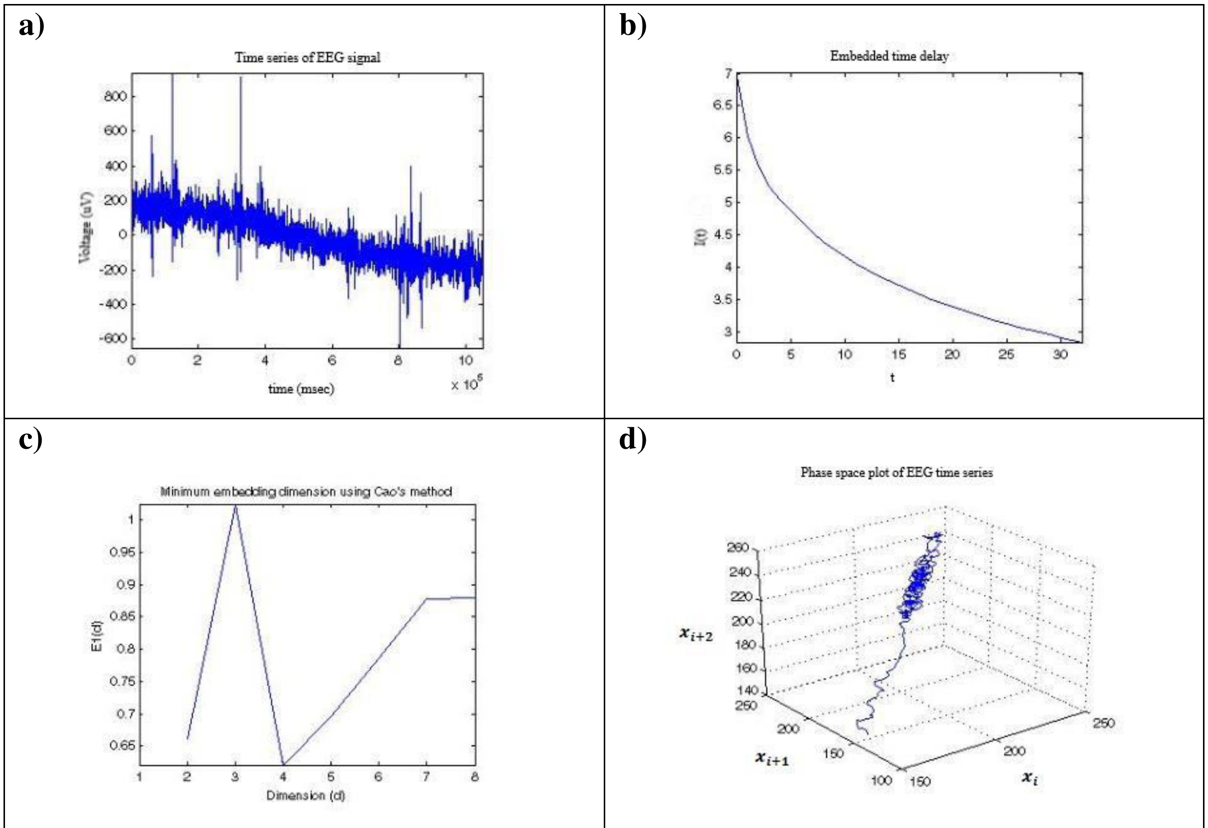

Figure 2: a) time series of EEG signal, b) embedded delay, c) embedded dimension and d) phase space plot

5. Conclusions:

Nonlinear time series analysis of EEG signals is studied by reconstructing its phase space using embedding theorem. This analysis concludes that phase space trajectory of EEG signals have chaotic nature. Further, we are interested in applying these techniques to the wide variety of brain signals like epileptic, sleep disorder and stroke patients. Classification of these signals creates clear understanding of pathological evidence of brain's health. One can easily find the transitions of signal variations in ictal periods. Identifying the origins of epileptic regions helps in better medication before surgical operations. In comparison between phase space plots, time series analysis of EEG signals is purely chaotic in nature. Variability of nonlinear parameters helps in recognizing the signals emerging, due to dysfunctions with in the brain.

Compliance with Ethical Standards

Conflict of Interest Statement

The authors report no conflicts of interest.

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Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors. The data used in the analysis is open source, which is taken from the reference [27-28].

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