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CHANNEL ESTIMATION DESIGN IN MULTI CELL MULTIUSER MIMO OFDM SYSTEMS WITH SIMULATION RESULTS

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ABSTRACT: This project investigates the uplink transmission in multi-cell multi-user multiple input multiple output (MIMO) orthogonal frequency division multiplexing (OFDM) systems. The system model considers imperfect channel estimation, pilot contamination (PC), multiple sub-carriers and multi-path channels. It is proposed a simple H-inf channel estimation that achieves good suppression to PC. The approach exploits the space-alternating generalized EM (SAGE) iterative process to decompose multi-cell multi-user MIMO problem into a series of single-cell single-user SISO problems, which reduces the complexity drastically. Analysis on mean square error (MSE) of H-inf in the presence of PC is also presented. The numerical results show that increasing the number of pilot subcarriers cannot mitigate PC, and a clue for relieving PC can be obtained. The H-inf realizes better suppression to PC than the LS and ML algorithms. Its performance is close to the optimal MMSE algorithm and can be improved as the increase in the length of channel impulse response (CIR). By using the SAGE process, the performance of the H-inf does not degrade in case of a large number of antennas at base station (BS).

INTRODUCTION

Wireless communications require the outstanding capability to combat multipath fading and to offer high spectral efficiency. Multiple-input multiple-output (MIMO) combined with orthogonal frequency-division multiplexing (OFDM) has been widely considered to be a promising candidate. Unlike the point-to-point MIMO, a multiuser MIMO (MU-MIMO) system that has low cost in terminals and better tolerance to wireless propagation

environment has been considered for future wireless communications [3]. In a multi cell scenario, it is well known that accurate channel state information (CSI) is critical for achieving high system performance. Since the mobility of users and the limited bandwidth, it is not possible to allocate dedicated pilots for the users in each cell, and therefore, the reuse of pilots is a must for users in different cells.

1.1.MIMO (MULTIPLE INPUT MULTIPLE OUTPUT)

Multiple-input and multiple-output is a method for multiplying the capacity of a radio link using multiple transmit and receive antennas to exploit multipath propagation. At one time in wireless the term “MIMO” referred to the mainly theoretical use of multiple antennas at both the transmitter and the receiver. In modern usage, “MIMO” specifically refers to a practical technique for sending and receiving more than one data signal on the same radio channel at the same time via multipath propagation as shown in Fig 1.1. MIMO is fundamentally different from smart antenna techniques developed to enhance the performance of a single data signal, such as beam forming and diversity.

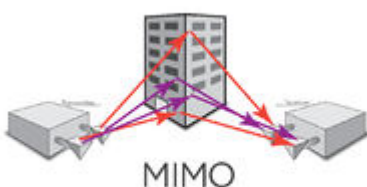


Fig.1.1 Multiple Input Multiple Output

MIMO can be sub-divided into three main categories, pre coding, spatial multiplexing or SM, and diversity coding. Pre coding is multi-stream beam forming, in the narrowest definition. In more general terms, it is considered to be all spatial processing that occurs at the transmitter. In (single-stream) beam forming, the same signal is emitted from each of the transmit antennas with appropriate phase and gain weighting such that the signal power is

maximized at the receiver input. The benefits of beam forming are to increase the received signal gain - by making signals emitted from different antennas add up constructively - and to reduce the multipath fading effect. In line-of-sight propagation, beam forming results in a well-defined directional pattern. However, conventional beams are not a good analogy in cellular networks, which are mainly characterized by multipath propagation. When the receiver has multiple antennas, the transmit beam forming cannot simultaneously maximize the signal level at all of the receive antennas, and precoding with multiple streams is often beneficial. Note that precoding requires knowledge of channel state information (CSI) at the transmitter and the receiver. Multiple input, multiple output-orthogonal frequency division multiplexing (MIMO-OFDM) is the dominant air interface for 4G and 5G broadband wireless communications. It combines multiple input, multiple output (MIMO) technology, which multiplies capacity by transmitting different signals over multiple antennas, and orthogonal frequency division multiplexing (OFDM), which divides a radio channel into a large number of closely spaced sub channels to provide more reliable communications at high speeds. Research conducted during the mid-1990s showed that while MIMO can be used with other popular air interfaces such as time division multiple access (TDMA) and code division multiple access (CDMA), the combination of MIMO and OFDM is most practical at higher data rates.

II..MIMO (MU-MIMO):

Multi-user MIMO (MU-MIMO) can leverage multiple users as spatially distributed transmission resources, at the cost of somewhat more expensive signal processing. In comparison, conventional, or single-user MIMO considers only local device multiple antenna dimensions. Multi-user MIMO algorithms are developed to enhance MIMO systems when the number of users or connections is greater than one. Multi-user MIMO can be generalized into two categories: MIMO broadcast channels (MIMO BC) and MIMO multiple access channels (MIMO MAC) for downlink and uplink situations, respectively. Single-user MIMO can be represented as point-to-point, pair wise MIMO. To remove ambiguity of the words receiver and transmitter we can adopt the terms access point (AP; or, base station), and user. An AP is the transmitter and a user is the receiver for downlink environments, whereas an AP is the receiver and a user is the transmitter for uplink environments. Homogeneous networks are somewhat freed from this distinction.

III.EXISTING SYSTEM:

This project considers a multi-cell multiple antenna system with precoding used at the base stations for downlink transmission. Channel state information (CSI) is essential for precoding at the base stations. An effective technique for obtaining this CSI is time-division duplex (TDD) operation where uplink training in conjunction with reciprocity simultaneously provides the base stations with downlink as well as uplink channel estimates. This project

mathematically characterizes the impact that uplink training has on the performance of such multi-cell multiple antenna systems. When non-orthogonal training sequences are used for uplink training, the project shows that the precoding matrix used by the base station in one cell becomes corrupted by the channel between that base station and the users in other cells in an undesirable manner. This project analyzes this fundamental problem of pilot contamination in multi-cell systems. Furthermore, it develops a new multi-cell MMSE-based precoding method that mitigates this problem. In addition to being linear, this precoding method has a simple closed-form expression that results from an intuitive optimization. Numerical results show significant performance gains compared to certain popular single-cell precoding methods. This project considers a multi-cell multiple antenna system with precoding at the base stations for downlink transmission. To enable precoding, channel state information (CSI) is obtained via uplink training. This project mathematically characterizes the impact that uplink training has on the performance of multi-cell multiple antenna systems. When non-orthogonal training sequences are used for uplink training, it is shown that the precoding matrix used by the base station in one cell becomes corrupted by the channel between that base station and the users in other cells. This problem of pilot contamination is analyzed in this project. A multi-cell MMSE-based precoding is proposed that, when combined with frequency/time/pilot reuse techniques,

mitigate this problem.

IVPROPOSED SYSTEM:

In statistics and signal processing, a minimum mean square error (MMSE) estimator is an estimation method which minimizes the mean square error (MSE), which is a common measure of estimator quality, of the fitted values of a dependent variable. In the Bayesian setting, the term MMSE more specifically refers to estimation with quadratic cost function. In such case, the MMSE estimator is given by the posterior mean of the parameter to be estimated. Since the posterior mean is cumbersome to calculate, the form of the MMSE estimator is usually constrained to be within a certain class of functions. Linear MMSE estimators are a popular choice since they are easy to use, calculate, and very versatile. It has given rise to many popular estimators such as the Wiener-Kolmogorov filter and Kalman filter. In many real-time application, observational data is not available in a single batch. Instead the observations are made in a sequence. A naive application of previous formulas would have us discard an old estimate and recompute a new estimate as fresh data is made available. But then we lose all information provided by the old observation. When the observations are scalar quantities, one possible way of avoiding such re-computation is to first concatenate the entire sequence of observations and then apply the standard estimation formula as done in Example 2. But this can be very tedious because as the number of observation increases so does the size of the matrices that need to be inverted

and multiplied grow. Also, this method is difficult to extend to the case of vector observations. Another approach to estimation from sequential observations is to simply update an old estimate as additional data becomes available, leading to finer estimates. Thus a recursive method is desired where the new measurements can modify the old estimates. Design and analysis of space-alternating generalized expectation-maximization-based h-inf algorithm in multicell multiuser multiple-input multiple-output systems. Earlier, we have shown that the MMSE algorithm can obtain optimal performance by using prior information and better suppression to PC. Although the use of SVD of channel correlation matrix is able to reduce the number of multiplications with negligible performance loss, its complexity is still quite high since obtaining the SVD itself has high computational complexity on the order of $O(N^3)$. Here, we introduce the H-inf algorithm, which were proposed in and to multicell MU-MIMO systems.

4.1 H-INF CHANNEL ESTIMATION

H-infinity loop-shaping is a design methodology in modern control theory. It combines the traditional intuition of classical control methods, such as Bode's sensitivity integral, with H-infinity optimization techniques to achieve controllers whose stability and performance properties hold good in spite of bounded differences between the nominal plant assumed in design and the true plant encountered in practice. Essentially, the control system designer describes the desired responsiveness and noise-

suppression properties by weighting the plant transfer function in the frequency domain; the resulting 'loop-shape' is then 'robustified' through optimization. Robustification usually has little effect at high and low frequencies, but the response around unity-gain crossover is adjusted to maximize the system's stability margins[14]. H-infinity loop-shaping can be applied to multiple-input multiple-output (MIMO) systems.. H-infinity loop-shaping has been successfully deployed in industry Easy to apply – commercial software handles the hard math.

- Easy to implement – standard transfer functions and state-space methods can be used.
- Plug and play – no need for re-tuning on an installation-by-installation basis.

As an alternative to the classical MMSE estimation, an H-inf filter can achieve an acceptable estimation performance without accurate knowledge of the statistical information of the involved signals. The idea of the H-inf filtering is to construct a filter that guarantees the H-inf norm of the estimation error is less than a prescribed positive value As for multicell MU-MIMO systems, the idea of the H-inf is to find an estimation method so that the ratio between the whole channel estimation error (between the j th BS and K users in each cell) and the input noise/interference is less than a prescribed threshold. Given a positive scalar factor s , the H-inf estimator for each received OFDM symbol needs to satisfy the following objective function

$$Z_j^{sup} = \frac{\|C_j^{\wedge} - C_j\|_W^2}{\|z_j\|^2} < s$$

Where $C_j^{\wedge} - C_j = (C_j^{\wedge} - C_j)HW(C_j^{\wedge} - C_j)$; C_j^{\wedge} is a $LQK \times 1$ vector, denoting the channel response vector to be estimated; $C_j = [CT_j1, \dots, CT_jQ]T$; $C_jq = [CT_jq1, \dots, CT_jqK]T$; and $W > \mathbf{0}$ is a weighting matrix. The H-inf channel estimation in multi cell MU-MIMO systems can be described as

$$C_j^{\wedge} = \eta_j \varepsilon_j^{-1} T^+ Y_j$$

Where $T = [T_1, \dots, T_Q]$, $T_q = [T_{q1}, \dots, T_{qK}]$, $T_{qk} = X_{qk} F_{N,L}$, and $\varepsilon_j = M_{1,1} + M_{1,2} \xi_j$ and $\eta_j = M_{2,1} + M_{2,2} \xi_j$, are both $LQK \times LQK$ matrices. ξ_j is a $LQK \times 1$ vector, satisfying $\|\xi_j\|_{\infty} = \max(|\xi_1|, \dots, |\xi_{LQK}|) < 1$, and $\xi_1 = \dots = \xi_{LQK}$. $M_{1,1}$, $M_{1,2}$, and $M_{2,1}$, $M_{2,2}$ can be expressed as

$$M_{1,1} = \Omega R^{\frac{1}{2}} + R^{-\frac{1}{2}}$$

$$M_{1,2} = s^{-\frac{1}{2}} \Omega W^{\frac{1}{2}}$$

$$M_{2,1} = \Omega R^{\frac{1}{2}}$$

$$M_{2,2} = s^{-\frac{1}{2}} \Omega W^{\frac{1}{2}} - s^{\frac{1}{2}} W^{\frac{1}{2}}$$

where $R = T^+ T = I_{LQK}$ if QPSK is adopted, $\Omega = \Omega_1 \Omega_2^{-1/2} - \Omega_2$, $\Omega_2 = (R - s^{-1} W)^{-1}$, and Ω_1 can be easily obtained by the canonical factorization of $I_{LQK} + \Omega_2$.

4.2 H-INF CHANNEL ESTIMATION VIA SAGE PROCESS

A direct solution to (13) will result from intense calculation of the matrix inversion and multiplication operations for each OFDM symbol of all users in Q cells over L paths, and the complexity is on the order of

$O(L^3Q^3K^3)$. In the case of large values of L , K , and Q , computational complexity load will be high. In multicell MU-MIMO systems, propagation vectors between the BS antenna arrays and different terminals often could be considered uncorrelated [4]. Since the SAGE can decompose the spatially multiplexed channels, we can apply this iterative algorithm to deal with the problem of high complexity. Generally, the SAGE process is developed to avoid matrix inversion of the ML estimator; therefore, we first assess the feasibility by applying SAGE. the above Equation can be rewritten as follows.

$$\begin{aligned} C_j^\wedge &= \eta_j \varepsilon_j^{-1} T^\dagger Y_j \\ &= \gamma C_j^\wedge{}^{ML} \end{aligned}$$

The numerator of equation is considered to be the whole estimation error between the j th BS and K users in each cell. Thus, the denominator of equation will be AWGN Z_j . However, if the local estimation error is considered, (e.g., between the j th and K users in the q th cell), the signal, except for that from the q th cell, will be the interference, which will finally change the establishment of the objective function. Where $\gamma = \eta_j \varepsilon_j^{-1}$. Equation can be interpreted as a filter matrix γ applied to the ML estimation, indicating some links between the H-inf and ML estimators. Thus, we can develop an H-inf estimator by combining the SAGE process. Instead of solving directly, the SAGE algorithm converts a multicell MU-MIMO channel

estimation problem into a series of single-cell single-user SISO channel estimation problems, making the dimensions of Ω , W , and R involved in the computation of ε_j , η_j much smaller. Thus, the calculation is simplified drastically. The SAGE-based H-inf estimation can be iteratively implemented as follows;

Initialization:

$$\begin{aligned} \text{For } q &= 1, \dots, Q, \\ \text{For } k &= 1, \dots, K \end{aligned}$$

$$Y_{jqk}^\wedge{}^{(0)} = T_{qk} \varepsilon_{jqk} \eta_{jqk}^{-1} C_{jqk}^\wedge{}^{(0)}$$

Where ε_{jqk} and η_{jqk} of dimension $L \times L$ are the simplified versions of ε_j and η_j , respectively. The initial value of channel estimation C_{jqk} is 1_L , where 1_L is an $L \times 1$ vector whose elements are all 1. by using iterations... finally solving equation is

$$\begin{aligned} C_{jqk}^\wedge{}^{(i+1)} &= \eta_{jqk} \varepsilon_{jqk}^{-1} T_{qk}^\dagger Y_{jqk}^\wedge{}^{(i)} \\ Y_{jqk}^\wedge{}^{(i+1)} &= T_{qk} \varepsilon_{jqk} \eta_{jqk}^{-1} C_{jqk}^\wedge{}^{(i+1)} \end{aligned}$$

while for $1 \leq k \leq K$ and $k _ = k$

$$Y_{jqk}^\wedge{}^{(i+1)} = Y_{jqk}^\wedge{}^{(i)}$$

4.3 PERFORMANCE ANALYSIS

Analysis of Matrix γ : To find a solution for the H-inf, we assume $R - s^{-1}W > 0$ where R is an identity. Matrix because QPSK is adopted, s is a positive scalar factor, and W is also a diagonal matrix that have equal

dimensions. Thus, $M_{1,1}$, $M_{1,2}$, $M_{2,1}$, and $M_{2,2}$ are all diagonal matrices, respectively. Finally, matrix γ is a real diagonal matrix with equal diagonal elements. Since the diagonal matrix γ is needed to estimate the performance of the H-inf, we will find the relation between γ and the identity matrix. First, it is assumed that

$$\gamma < I_{LQK}.$$

Note that R will not be an identity matrix if 16-QAM, 64-QAM, or other modulations are adopted. However, γ is always a diagonal matrix. The proposed algorithm is valid for the different modulations. To satisfy equation, one has $\varepsilon - \eta > 0$. By applying the above equation, we can get

$$\begin{aligned} \varepsilon - \eta &= (M_{1,1} + M_{1,2}\xi_j) - (M_{2,1} + M_{2,2}\xi_j) \\ &= R^{-\frac{1}{2}} + s^2 W^{-\frac{1}{2}} \xi_j > 0. \end{aligned}$$

Therefore, our hypothesis is valid. Intuitively, when W is fixed, a smaller s is made, a smaller γ is obtained, and a better performance is achieved, which is the intrinsic characteristic of the H-inf algorithm, as will be discussed in the following.

4.4 IMPACT OF PC ON H-INF

Since the estimation errors in cells are independent of each other, we analyze the channels from the K users in the j th cells. The following assumptions are made: 1) All subcarriers have equal power; 2) phase-shift orthogonal pilot sequences are used for different users within each cell; and the same pilot sequences are reused in other

cells.

The channel estimation of the H-inf can be rewritten as

$$\begin{aligned} \hat{C}_{jj}^{H-inf} &= \gamma T_j^\dagger Y_j \\ &= \gamma T_j^\dagger \sum_{q \neq j}^Q T_q C_{jq} + \gamma C_{jj} \\ &\quad + \gamma T_j^\dagger Z_j. \end{aligned}$$

The MSE expression of the H-inf algorithm for multicell MU-MIMO systems in the presence of PC is given as follows:

$$\begin{aligned} \text{MSE}_{H-inf} &= \frac{1}{L} r_{nn}^2 \sum_{q \neq j}^Q d_{jq} + \frac{1}{L} r_{nn}^2 \sigma^2 + \\ &\quad \frac{1}{L} (1 - r_{nn})^2 \end{aligned}$$

4.5 COMPLEXITY ANALYSIS

Considering the number of complex multiplications for each OFDM symbol as a complexity Metric, the inversion of an $n \times n$ matrix requires n^3 operations, the pseudo inverse of an $n \times r$ matrix requires $2r^2n + r^3$ operations, and the product of an $m \times r$ matrix with an $r \times n$ matrix requires $m \cdot n$ operations. Let K_{it} denote the number of iterations that should not be too large due to the superior convergence property of SAGE. A comparison of complexity between the LS, MMSE, and proposed H-inf algorithms is given in Table I. As expected, the H-inf estimation has less complexity than the MMSE algorithm, and the complexity can be further reduced by using the SAGE iterative process

Algorithm	Number of operations per OFDM symbol
LS	$(2(LK)^2 + (LK))N + (LK)^3$
MMSE	$(2(LK)^2 + (LK))N + (LK)^3 + 2N^3 + LN^2 + LN$
H-inf	$(3(LK)^2 + LKQ)N + 3(LKQ)^3$
SAGE-based	$K_{it}((3L^2 + L)N + 3L^3)$

TABLE-4.1.COMPLEXITY OF CHANNEL ESTIMATION ALGORITHMS

V.SIMULATION RESULTS:

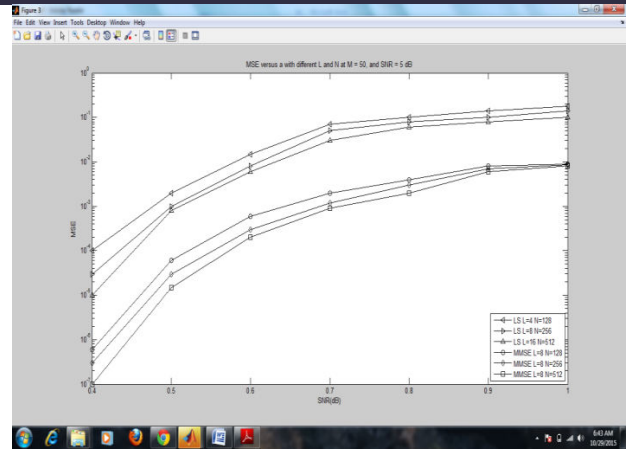


Fig.5.3 MSE versus α with different L and N at $M = 50$, and $SNR = 5$ dB.

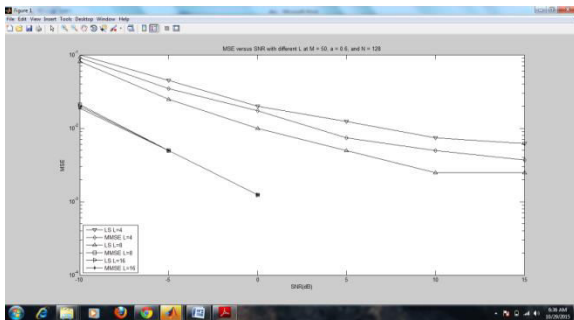


Fig.5.1 MSE versus SNR with different L at $M = 50$, $\alpha = 0.6$, and $N = 128$.

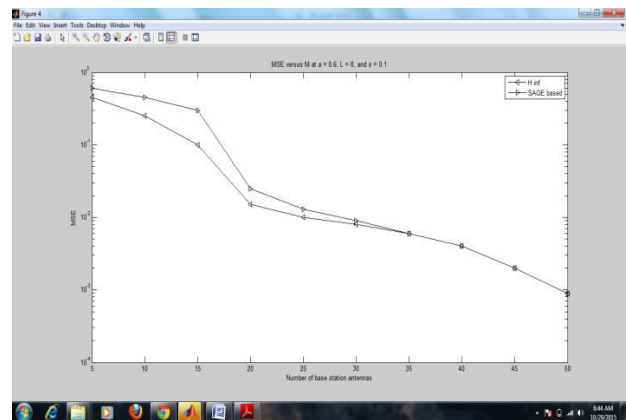


Fig.5.4 MSE versus M at $\alpha = 0.6$, $L = 8$, and $s = 0.1$

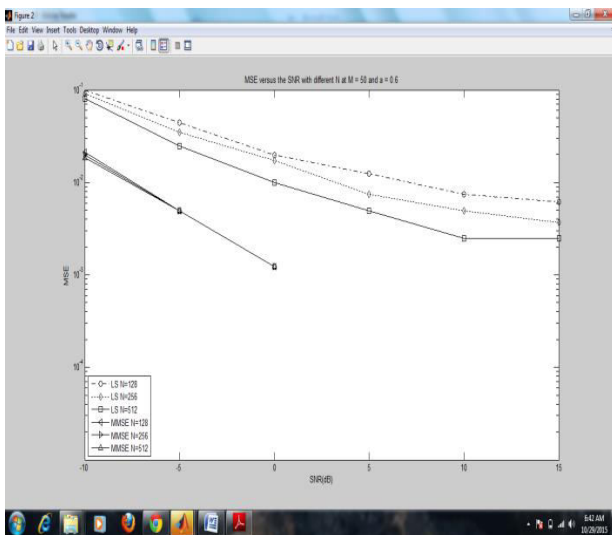


Fig.5.2 MSE versus the SNR with different N at $M = 50$ and $\alpha = 0.6$.

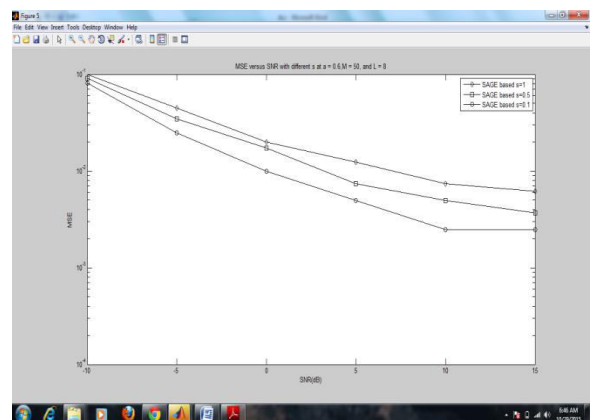


Fig.5.5 MSE versus SNR with different s at $\alpha = 0.6$, $M = 50$, and $L = 8$

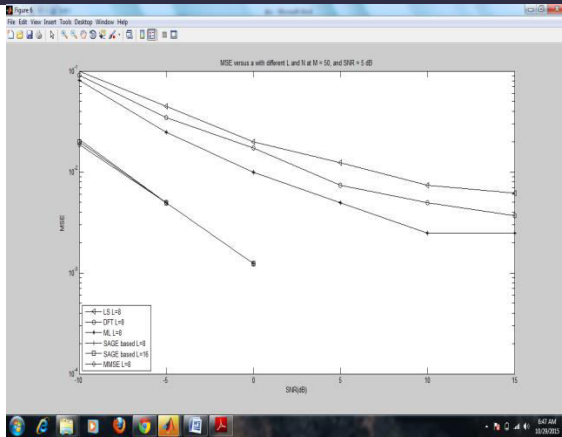


Fig.5.6 MSE versus SNR with different L at $a = 0.6$, $M = 50$, and $s = 0.1$.

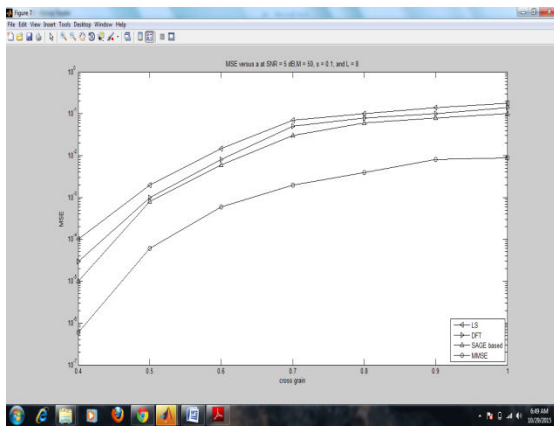


Fig.5.7 MSE versus a at SNR = 5 dB, $M = 50$, $s = 0.1$, and $L = 8$.

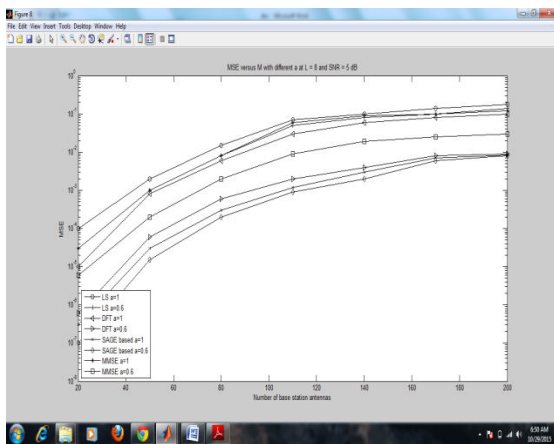


Fig.5.8 MSE versus M with different a at $L = 8$ and SNR = 5 dB

CONCLUSION

It has been analytically investigated the impact of PC on the several pilot-based channel estimation algorithms, including classical LS, MMSE algorithms, and our proposed H-inf algorithms in multicell MU-MIMO systems under a realistic system model that considers imperfect channel estimation, PC, multicarrier, and multipath channels. Analytical expressions were derived, and comparisons were made. It has been shown that, of all the algorithms, the optimal MMSE is most resistant to PC with high complexity. By slightly increasing the number of OFDM subcarriers, PC suppression can be achieved in the MMSE. In addition, by increasing the number of pilot subcarriers for all channel estimation algorithms, PC cannot be mitigated. For the proposed H-inf algorithms, proper length increment of CIR is helpful for the suppression of PC. Simulation results have shown that the proposed H-inf algorithm has almost the same performance as MMSE, and it leads to better suppression to PC than LS, DFT, and ML. In addition, the H-inf via the SAGE iterative process does not introduce any performance loss when the number of antennas is large at each BS in multi cell MU-MIMO systems.

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