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Study of Blood Flow Into The Capillaries Through Viscoelastic Fluid Model

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ABSTRACT :

The propagation of waves into the arteries were explained by interaction between fluid structure for this different models were used in the section of arteries which cylindrical geometry by allowing axially symmetric blood flow in the current paper we described a simples, easy and closed reduced model which accounts for viscous flow of blood and the newly obtained fluid interaction structure is of hyperbolic and parabolic type the walls of arteries were modeled through the modeled of Koiter Shell linearly viscoelastic cylindrical model and stated the blood flow through uncompressible viscous equation which gives the relation between stress coupled and stress resultant. In this paper we studied the relationship between stress and strain which was corresponding with the shell model of Koiter in two forms that is linearly and cylindrical viscoelastic. Shell model by giving the effective equations.

Keywords: Stress Resultant, Uncompressible, Viscous Flow, Cylindrical Geometry.

INTRODUCTION:

Today the study of flow of viscous fluid by tube like structure included arteries give special attention due to its interest of many researchers in various applications. The important application is the flow of blood through arteries of human by understanding the propagation of blood waves into the walls of arteries; proper heamodynamics is the major important mechanism which leads to different problems in the cardiovascular function. It is approved that from medium size to large size arteries blood is considered as the thick, sticky, uncompressible fluid although it is the mixture of red and which blood cells and plasma platelets having unnewtonian nature. The study of coupling between vessels wall motion and flow of pulsatile blood a deep explanation of vessels wall biomechanical features reasonable for mathematical expressive and numerical problems having complexity is far of current abilities of computer. The un linearity of the interaction between fluid structure so strict that although easy explanation of the mechanism of vessels wall considering homogenous, linearly elastic behaviour caused to hard numerical algorithms with computational stability and converging properties to plan a mathematical model which will caused to problems and gives open suggestion to methods of numerical generating hard solution in the short time duration. Different simplifications are necessary to be introduced which can be based on different simplifying models which include two dimensional &three dimensional models which described. Fluid structure interactions among the uncompressible flow of viscous fluid and its motion in the linearly

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elastic cylindrical membrane, we also introduced which were much complex. Often time the terms like viscoelastic nature are also added with the walls of vessel model to enhance stability and convergence of basic numerical algorithm [3,9] and also for giving regularity in the existence of its convergence.
In fact the current review on good posedness of fluid structure relation between viscous incompressible fluid and its viscoelastic structure which include many additional simplifying features. About Koiter Shell model the description was also stated by Ciarlet and Lods [7] and through this model they show that the model is same asymptotic behaviour with three dimensional membrane model the two sub-models of Koiter Shell model which give different expressions of Cylindrical geometry and extended linearly elastic Koiter model by showing viscous effect and also give some measurement of features of mechanics of walls of Vessel [19,17,13]

Objective :

i) To investigated new simple closed reduced model which accounts for the viscous fluid dissipation.
ii) To maintain the interactions between fluid structure systems.
iii) To measure arterial stress strain relationship.
iv) To derived an effective three dimensional model for observing the mechanical properties of vessel wall.
v) To find the nature of model on the basis of shearing rate.

Application :

1. This model useful for showing that how the leading order viscous fluid dissipation imparts long term viscoelastic effect on the motion of the vessel walls.
2. It is also use full for showing the bending rigidity of vessels walls.
3. It is applicable for demonstrating the behaviour of underlying fluid structure interaction.

Mathematical Modelling and Consideration

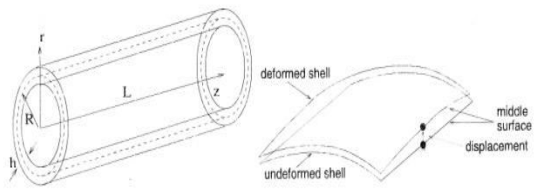
Koiter Viscoelstic Cylindrical Shell model
It includes two models.

a) Koiter linearly elastic Shell model.
b) Koiter linearly viscoelastic Shell model.

a) Koiter linearly elastic Shell model

Take a cylindrical Shell having the radius of internal area is equal to r=R with h and L are the thickness and length of that cylindrical Shell which follows the basic assumptions that

- The Shell must be thin
- The pulling force is not so high
- The stress is in the form of plane state



The expressive formulation of Koiter linearly elastic Shell model shows that there is a changes of in the energy of strain and its density function which based on fluctuations in tensors of curvature and metric surfaces which results into surface stretching and showing bending effects. That's why it is stated that the stretching and the bending of the Shell have ability to change in the strain energy which was only given by weak formulations of Koiter Shell. The displacement of the internal surface is given by

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£(z) = (£$_z$(z), £$_r$(z)) -------- (1)

Where,

£$_z$(z)= Longitudinal displacement

£$_r$(z)= radial displacement.

And the variation in the tensors of metric and curvature surfaces of the Koiter Shell model are described below.

$$V(£)=\begin{bmatrix} £_z' & O \\ O & R£r \end{bmatrix}$$

$$\rho(£)=\begin{bmatrix} -£_z'' & O \\ O & £_T \end{bmatrix}$$

Where (i) shows the derivatives of Z which introduced the space function.

$Vc = H_O^1$(O,L) x H_O^2(O,L) ------ (2)

The Koiter Shell of linearly elastic cylindrical model gives the weak for mulations in the form of n which stated as below.

Ŋ= (ŋ$_z$, ŋ$_r$) €Vc

Such that

$\int_o^L f\,.£Rdz,$ £€Vc---------------- (3)

Where dot (.) shows the scaler product.

A.B. = Tr(AB)T, A.B.€M$_2$ (R)≅ R^4

f = Surface density

A = Elasticity tensor which shows following

$$AE = \frac{4\lambda\mu}{\lambda+2\mu}(A^C.E)\, A^C + 4\mu A^C EA$$

$$A^C=\begin{bmatrix} 1 & O \\ O & R^2 \end{bmatrix},\quad A^C=\begin{bmatrix} 1 & O \\ O & 1/R^2 \end{bmatrix}$$

Where λ and μ are known as Lame constant which shows displacements

∴ the weak formulation stated.

$\int_O^L (f_z£_z + f_r£_r)d_z \quad \forall\, (£_z, £_r)$ €Vc

by using following quantities in the Lame constant and modulus of elasticity E and Possion's ratio 6 We can written as

$$\frac{Z\mu\lambda}{\lambda+Z\mu}+2\mu=4\mu\frac{\lambda+\mu}{\lambda+2\mu}=\frac{E}{1-6^2}$$

$$\frac{Z\mu\lambda}{\lambda+Z\mu}+4\mu=4\mu\frac{\lambda+\mu}{\lambda+2\mu}\,\frac{1}{2}\,\frac{\lambda}{\lambda+\mu}=\frac{E}{1-6^2}\,6$$

And the A is given as

$$AE=\frac{2E6}{1-6^2}(A^C.E)A^C+\frac{2E}{1+6}A^C EA^C,\ E€VR^2$$

From above expression we also get weak formulation

$\int_O^L (f_z£_z + f_r£_r)d_z \quad (£_z, £_r)$ €Vc

∴ the static equilibrium equations is given by in the form of Fz, and Fr

$$Fz=\frac{-hE}{1-6^2}(ŋ_z''+6\frac{1}{R}ŋ_r')$$

$$Fr=\frac{-hE}{1-6^2}\left(6ŋ_z''+\frac{ŋr}{R}\right)+\frac{h^3E}{12\,(1-6^2)}(ŋ_r'''-26\frac{1}{R^2}ŋ_r''+\frac{1}{R^4}ŋ_r'')$$

By using above equation we can study about the pulsatile blood flow in the arteries for this work we consider that in vive before arteries are stretch under the load of internal pressure that the walls of arteries are longitudinally binds such that the longitudinal displacement are too much less. [6] [12] which described below.

$$\left(\frac{hE}{R(1-6^2)}+Pref\ 6{,}12\right)\frac{nr}{R}+\frac{h^3E}{12(1-6^2)}\left(ŋ_r'''-26\frac{1}{R^2}ŋ+\frac{1}{R^4}ŋr\right)=fr$$

This equation is get from weak formulation that considering n$_2$=O &

V_c^o= V$_C$∩ $\{€_z = o\}$

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b) Koiter linearly Viscoelastic Shell model

The "Stress resultant" shows the relation between stress and strain which gives the force of internal surface with the variation in metric tensor similarly, the stress couples shows the moment of Bending by showing variation in the tensor of curvature [14] this terms can be get from the equation of Koiter linearly cylindrical Shell model of equation (3) and the gradient of the Stored energy functions can be obtained by using following assumptions with their equations. Force of internal surface which is also called stress resultant for the elastic Koiter Model is given by

$$N := \frac{h}{2} Ar(\eta) = \frac{h}{2}\begin{bmatrix} \frac{2E6}{1-6^2}\frac{nr}{R} & O \\ O & \frac{2E}{1-6^2}\frac{nr}{R^3} \end{bmatrix}$$

Moment of Bending for elastic Koiter Shell Model.

$$M := \frac{h^3}{24} A\rho(n) = \frac{h^3}{24}\begin{bmatrix} \frac{-2E}{1-6^2} n_r^{11} + \frac{2E6}{1-6^2}\frac{nr}{R^2} & O \\ O & \frac{2E}{1-6^2}\frac{nr}{R^4} \frac{2E6}{1-6^2}\frac{1}{R^2} n_r^{11} \end{bmatrix}$$

By above equation we get the effects of before stress by showing the resultant of stress [20] with reference to pressure [6-12] along with external strain [3,5,11].

$$\frac{h}{2} N_{ref} = hRA^c \begin{bmatrix} O & O \\ O & Pref\,\frac{R}{h}\eta r \end{bmatrix} A^c$$

- The resultant of stress for before stressed elastic Koiter Shell.

$N := \frac{h}{2} Ar\,(\eta) + \frac{h}{2}\, Nref$

- The stress resultant for viselastic prestressed Koiter Shell

$N := \frac{h}{2} AT\,(\eta) + \frac{h}{2} Br\,(\eta) + \frac{h}{2}\, Nref$

- Stress coupled for viscoelastic Koiter Shell

$M := \frac{h3}{24} A\rho\,(n) + \frac{h3}{24} \beta\rho\,(n)$

Where η is the time derivatives & β is given by

$$\beta E = \frac{4\lambda v \mu v}{\lambda v + 2\mu v}(A^C.E)A^C + 4\mu_v A^C E A^C$$

Where μv, λv are the viscous derivatives of Lame constant μ & λ

$$Cv := \frac{2\lambda v \mu v}{\lambda v + 2\mu v} + 2\mu v$$

$$Dv := \frac{2\lambda v \mu v}{\lambda v + 2\mu v} + 2\mu v$$

From all above equation we get the linearly viscoelastic cylindrical pre-stressed Koiter Shell model having longitudinal zero displacement.

$$fr = \rho wh\frac{\lambda^2 \eta r}{dt^2} + Co\eta r - C_1\frac{d^2\eta r}{dz^2} + C_2\frac{d^4\eta r}{dz^4}\, D_0\frac{d\eta r}{dt} - D_y\frac{d^3\eta r}{dtdz^2} + \frac{d^5\eta r}{dtdz^4}$$

Where ρω = density of Shell

$$C_o = \frac{h}{R^2}\frac{E}{1-6^2}\left(1 + \frac{h^2}{12R^2}\right) + \frac{Pref}{R}$$

$$C_1 = 2\frac{h^3}{12R^2}\frac{E6}{1-6^2}$$

$$C_2 = \frac{h^3}{12}\frac{E6}{1-6^2}$$

$$D_o = \frac{h}{R^2} Cv\left(1 + \frac{h^2}{12R^2}\right)$$

$$D_1 = 2\frac{h^3}{12R^2} D$$

$$D_2 = \frac{h^3}{12} Cv$$

This equation gives the relation between time dependant fluid flow driven by pulsatile inlet and outlet pressure to simplify nomination we can denote n for radial displacement nr.

Result & Discussion :

To study the coupling between the motion of vessels wall and pulsatile blood flow. The current research is help full a deep description of properties of arterial walls include homogeneity of material with less displacement and small deformation which leads to linear elasticity. A common set of special features leads to simplifying models includes small vessels walls thickness allowing a reduction from two dimensional shell models and cylindrical geometry of a section of an artery in the study it is observed that there is negative bending rigidity of arteries which reduces the Shell model to a membrane model and the simplifications include axial symmetry of loading exerted by the blood flow to the vessel wall the main result of this study is that by using the priori estimates based on an energy inequality, coupled with asymptotic analysis and homogenization theory, we develop effective closed fluid structure interaction model and capture viscoelastic properties of major arteries.

CONCLUSION:

In the current paper we explain easy and expressive closed model which shows the flow of blood by viscoelastic arteries in the form of cylindrical dimensions by considering axially symmetric flow by using Koiter linearly. Elastic Shell model and Koiter linearly viscoelastic Shell model we can deduce the equations which gives the relationship between blood flow pressure also gives the interaction between strain energy and stress resultant..

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