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NOMINAL BALLISTIC MISSILE ASCENT OPTIMIZATION WITH THERMAL INVESTAGATION

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Abstract

Ballistic Missile Trajectory is one of the big issues to simulate and also to be optimized. The main objective of this paper is to find a method to obtain the Nominal trajectory for the ballistic missile through finding out the optimal parameters for each ballistic trajectory phases. To define the problem; the paper will list all governing equations and describe the parameters of input and output for each phase of the trajectory. Then, a method to reduce the number of input parameters will be discussed. We study each parameter effect to get the most optimal trajectory for the missile. At the final, optimal flight will be discussed in details. Although, we investigate the thermal effect for entry point from space into the earth atmosphere

Keywords **Aerospace, Spacecraft, optimal, nominal, ascent, Ballistic, and missiles**

Nomenclature

γ	Inclination angle to the normal from center of the earth
Σ	Difference angle
θ	Angle
ρ	Injection radius to re-entry radius
A	Area
D	Diameter
Dist	Distance
e	Eccententry
F	Force
M	Mass
r	radius
T	Time
V	Velocity
X	Local X coordinate in the plane of trajectory
Z	Local Z coordinate in the plane of trajectory

Subscripts

burn	Burning
Cal	Calculated
Drag	Drag Effect
e	At the end of ballistic phase
Final	At the end of ballistic phase
i	Initial for ballistic phase
Impact	At the impact point
o	Initial
Pay	Payload
Pay	Payload
Prop	Propulsion
Propellant	Propellant
s	Structural
Structure	Structure
Tip	At the end of boost phase
Vac	Vacuum

Introduction

The more important start is just to specify what the ballistic missiles phases are and which one will be discussed here in this paper

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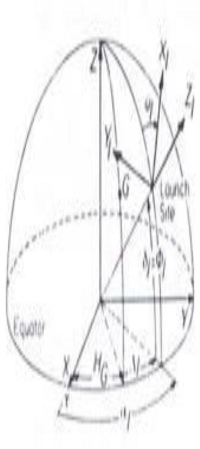

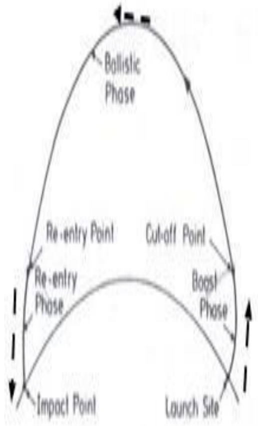

Fig.1 The launch site coordinate system Fig 2 Ballistic missile trajectory

A ballistic missile trajectory divided into three stages:

1. **The boost stage:** the vehicle is thrusted by one or more engines to get out the atmosphere layer. Guidance is Concerning, which will not be discussed here. the thrusted stage can be divided into: an open-loop phase and a closed-loop, or guidance phase. The open-loop phase is preprogrammed and consists of vertical direction lift-up, during this, the missile is rolled in thrust direction plane coincides with the target motion plane, and followed by a pitch-over and depends on gravity turn.
   The navigation computation instructions is based on the vehicle's actual location and speed. Coordinates of the target point characterize the subsequent phase of guidance. the air drag cannot be neglected during the last stage with respect to the gravity force, the free fail ascent phase follows the basic thrusted phase.
2. **The ballistic stage:** is the Big part of the trajectory and the vehicle enters the atmosphere at the end of it.
3. **The re-entry stage:** is the following passage downwards through the air until touch at the Earth surface.

Analysis and Design

From of the View Of 2D plane of missile Motion, we find the input parameters are needed for each phase and the output parameters are the given by equations that listed in Ref [1] and summarized as the following table ….

The main Input for the simulation is that located in the first column which divided into the following main objects:

1- Rocket Motor specifications Which include the first 4 parameters (Fvac, Ae, T_{burn}, $M_{Propellant}$)
2- Payload Mass
3- Main angles (γ_{Tip}, γ_{final})
4- Desired Range
5- Structure Mass

All of this input affect on the Range that actually covered by the missile in different ways. So, we must find a way to get the initial conditions for this input parameters as optimum as possible.

Parameters Reduction :

a. γ_{final} initial Value :

The injection flight path angle or γ_{final} for the boost phase must have an initial condition. Due to the following analysis, which start from the equations of the kepleriane ellipse equations of motion Ref[1] for more details.

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The analysis:

$$\Sigma = \theta_e - \theta_i$$
$$1 + e\cos(\theta_e) = \frac{P}{r_e}$$
$$1 + e\cos(\Sigma - \theta_i) = \frac{P}{r_e}$$
$$1 + e\cos(\theta_i)\frac{1-\tan^2\Sigma/2}{1+\tan^2\Sigma/2} - e\sin(\theta_i)\frac{2\tan\ \Sigma/2}{1+\tan^2\Sigma/2} = \frac{P}{r_e}$$

And with the help of equations that listed in Miller[3], we find…

$$Ki = \frac{Vi^2}{\mu/ri}$$
$$p = ri * ki * \cos^2(\gamma_i)$$
$$e = \sqrt{Ki(Ki-2)\cos^2(\gamma_i)+1}$$
$$r = \frac{P}{1+e*\cos(\theta)}$$

(2)

A simpler expression can obtain for $\tan(\frac{\Sigma}{2})$ and subsequent of the expression of p, $e * \cos(\vartheta i)$ which may can obtained from equation 2, yields..

$$\tan(\frac{\Sigma}{2}) = \cos(\gamma_i)\frac{ki*\sin\gamma i + \sqrt{k_i^2(1-\rho_i^2\cos^2\gamma i) - 2ki(1-\rho i)}}{2 - k_i\cos^2\gamma_i(1+\rho_i)}$$

(3)

Where ρ_I is the injection radius over re-entry radius ratio..

$$\rho_i = \frac{ri}{re}$$
$$Ve = Vcr\sqrt{Ki - 2(1-\rho_i)}$$
$$\cos(\gamma_e) = \frac{ri}{re}\frac{Vi}{Ve}\cos(\gamma_i)$$

(4)

(1)

The quadratic equation, Eq. (1), has two solving paramters. Eq. (3) solution matches to a calculated angles, **θe =θi +Σ** between 180o and 360o, which the re-entry interval true anomaly as known.

The re-entry stage paramters (velocity and direction angle) which are needed to the re-entry point determination, followed from the equations energy conservation and angular force which listed in equation (4.b) and (4.c).

The injection and re-entry altitude seems to be equal relative to the earth radius, so, a fair assumption of ballistic range angle and re-entry conditions is calculated by ρi=1, i.e. $r_i=r_e$.

Then the expression simplify, facilitating the analysis, from equation (3) we find for the range angle...

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$$\tan\frac{\Sigma}{2} = \frac{ki * \sin(\gamma_i)\cos(\gamma_i)}{1 - k_i \cos^2\gamma_i}$$

(5)

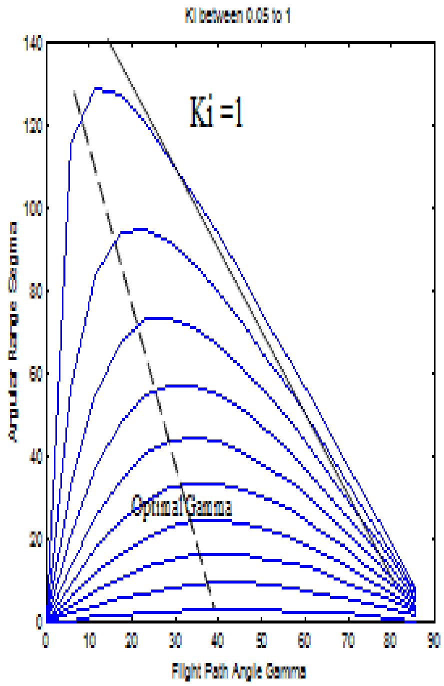


Fig 3 Angular Range Angle Relationship With Flight Path Angle

in Fig (1), shows that for Ki <1, i.e booster speed is less than circular speed, there is an optimized angle of the flight path, until maximum range.

For Ki =1 Eq. (5) simplified to…

$$\tan\frac{\Sigma}{2} = \cot\gamma$$

(6)

After this, the covered area decreases with flight angle linearly.

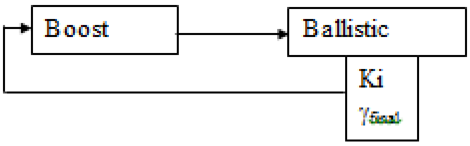
Given the injection conditions, Ki and γ_i , the range can be determined with Eqn. 5. In general, the angular range is known , at least approximately, and the injection conditions have to be determined as in our case. Solving Eqn.5 for $\cos^2\gamma$ leads to..

$$\cos^2(\gamma_i) = \frac{1}{2}\cos^2(\frac{\Sigma}{2})\left[1 \pm \sqrt{1 - 4\frac{1-Ki}{Ki^2}\tan^2(\frac{\Sigma}{2})}\right] + \frac{1}{Ki} si$$

(7)

So, from this equation we can determine the flight path angle due to knowing the Ki and the range. So, the only way to do this is the following steps..

- Estimate an initial γ_{final} for the boost phase.
- Get Ki from the first estimation
- Calculate γ_{final} From Eq. (7)
- Recalculate the boost phase Calculation with the new desired γ_{final}



b. Fvac Effect :

For a certain rocket motor specification, Ae, Mpropellant, and Tburn assumed to be const and the only variable change that the fuel that filled in the propellant tanks which affect in Fvac only.

So, here this paper will discuss the effect of Fvac change on the trajectory shape and range.

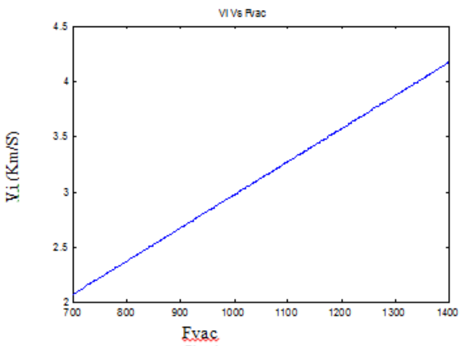


Fig 4 Vi Vs Fvac Variation

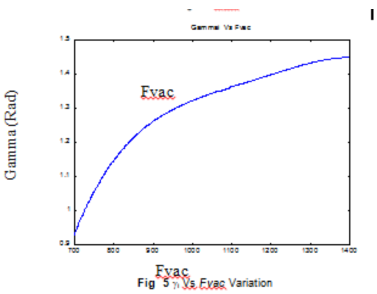


Fig 5 γ_i Vs Fvac Variation

In Fig (1), (2), we only change Fvac from 700 KN to 1400 KN with step 50 KN. Fig 1, shows that there are a approximate linear relation between V_i and Fvac.

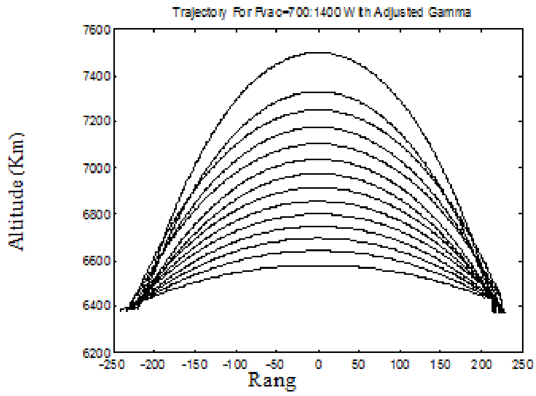


Fig 5 Trajectory For Fvac=700:1400 With Adjusted Gamma

For Fvac < 700, there no real solution for equation 7 because that the range is const. We note that as max γ_I that reached from Fvac 1400 KN is equal to 1.45 Rad which mean that is nearly to be horizontal flight γ_i =90o but because the tip-over angle, the missile can't reach to this angle. From Fig 4, we recognize that …

- The Difference in the actual range covered by the missile didn't change with high value (within 10-20 Km) when I change the Fvac to the max.
- The Distance covered in re-entry Phase as shown in Fig 5 is variant from long to short then to long again due to the variation of the Fvac.
- For Fvac < $Fvac_{Optimal}$, increasing in the distance that covered in Re-entry phase, the heating and the drag increase also which mean this is not an optimal solution.
- For Fvac > $Fvac_{Optimal}$, increasing the Re-entry Flight Path angleand decreasing in the distance that covered in Re-entry phase, the

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heating and the drag increase also which mean this is not an optimal solution.

- For Fvac that make Ki more than 2 which mean the Vi is a double of the circular velocity, the missile will not return to the surface again.

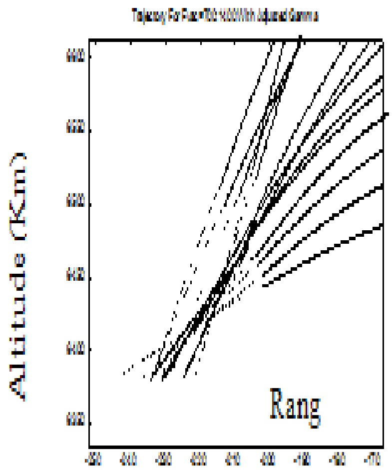

Fig 6 Trajectory For Fvac=700:1400 With Adjusted Gamma (zoomed)

The figure 2,3 makes me able to get a function fit the curve that I have for Fvac and Vi relation and Fvac and γ_I relation which equal to…

γ_I = -1.141043900302843e-015***Fvac**^4 + 5.055798324696411e-012***Fvac**^3- 8.313131361096544e-009***Fvac**^2 + 3.010603371978098e-003***Fvac**- 3.123098477538886e-002

Vi=-578089621976529e-012***Fvac**^4 + 3.044076174407523e-008 ***Fvac**^3- 5.282706789864383e-005***Fvac**^2 + 4.111018160088432e-002***Fvac**- 1.082346598610157e+001

Optimal Flight:

From Ref[1] and Ref[3], As clear from Fig (1,4 and 5), there is that the optimal boosted flight angle, which the angular angle is maximized for parameters (Vi, ri, and re) or, equivalently, the injection velocity is minimized for a given Σ, ri, and re.

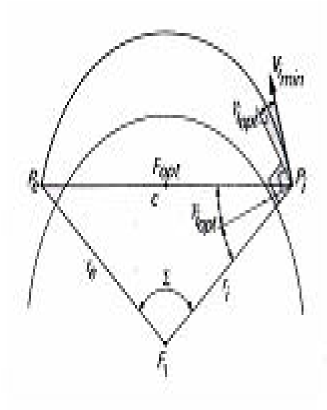

Fig 7 Optimal Flight Path Geometry

conditions derivation for the optimized flights using the usual visual method. The optimized flight trajectory is shown in Fig (6). which observed for a given range the boost velocity is a minimized if the 2 circles, with Pi and Pe as center and (2a-ri) and (2a-re) as radius.

The point of impact, Fopt is lied on the chord between Pi and Pe. So, for optimzed flights.

$$a_{min} = \frac{1}{4} * (c + r_i + r_e) \quad (8)$$

c is the length of the chord Pi-Pe, given by

$$c^2 = r_i^2 + r_e^2 - 2 r_i r_e \cos\Sigma \quad (9)$$

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The major axis (a) is a function of Ki and ri. Substitution of equation of (a) in equation (9) and Eq (9) in Eq (8) by an equation for the minimum boosted velocity ratio, Ki_{min}, the solution of which is

$$K_{i_{min}} = \frac{1-\rho_i - 2\sin^2 \Sigma/2 + \sqrt{(1-\rho_i)^2 + 4\rho_i \sin^2 \Sigma/2}}{\cos^2 \Sigma/2} \quad (10)$$

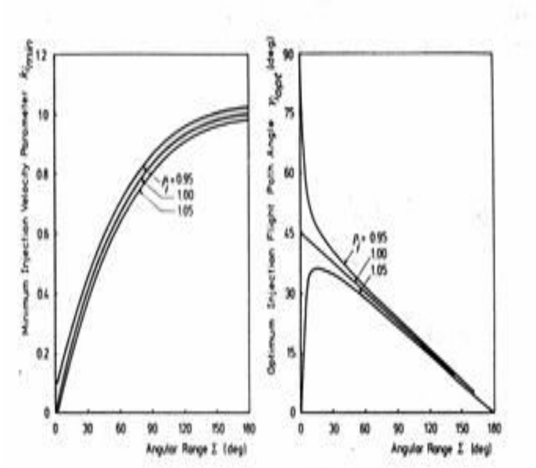

Fig 8 Angular Range With Optimal K

Fig (7) show the optimized flight angle which is calculated by finding that the angle Fopt_Pi_Fi equals double value of the flight angle. using the plane triangles on the Fi_Pi_Pe we can reach to

$$\tan^2 \gamma_{i_{opt}} = \frac{r_e^2 - (r_i - c)^2}{(r_i + c)^2 - r_e^2} \quad (11)$$

In Fig (7b), for a given **Case 1 range**, Eq (10 & 11) can be used for optimized boost conditions. For a given injection altitude value , the boosted velocity can be minimized.

These trajectories can be called also the minimum energy trajectories. These trajectories are not optimized from payload is point of view only, because the condition of the boosted altitude is fixed. determination process of the Max. payload path for a specfic missile, we need to identify the Charc. of the trusted trajectory (Boost Phase) from altitude, velocity and flight path angle relations.Max. payload paths with Min. boosted velocity paths will be same.

For a specific injection velocity & radius (**Case 2**) , the largest angular range can be calculated by solving Eq (10) for Σ. Through catching the maximum range for a given value of Ki

$$\tan^2 \frac{\Sigma_{max}}{2} = \frac{Ki}{2} \cdot \frac{Ki - 2(1-\rho_i)}{2 - Ki(1+\rho_i)} \quad (12)$$

In case of re-entry and injection altitude are equal, the equations for the min boosted velocity (Specific covered range, and max injection speed) and the optimal flight angle can be simplified as :

For Case 1:

$$K_{i\min} = \frac{2\sin \Sigma/2}{1+\sin \Sigma/2}$$

$$\tan \gamma_{i\,opt} = \left[\frac{1-\sin \Sigma/2}{1+\sin \Sigma/2}\right]^{1/2} \quad (13)$$

For Case 2:

$$\tan \frac{\Sigma_{\max}}{2} = \frac{Ki}{2\sqrt{1-Ki}}$$

$$\tan \gamma_{i\,opt} = \sqrt{1-Ki} \quad (14)$$

Therefore, using case 1, that the range is the input, we can get the Ki_{min} and γ_{I_opt}. These values can obtain from the first of the calculation because it's depended on Σ which known from the first of calculation. The previous note is valid only for the flight path angle but for Ki we must run the boost phase routine to get Ki then decrease/increase the Fvac depend on Ki calculated is greater/less than the Kmin.

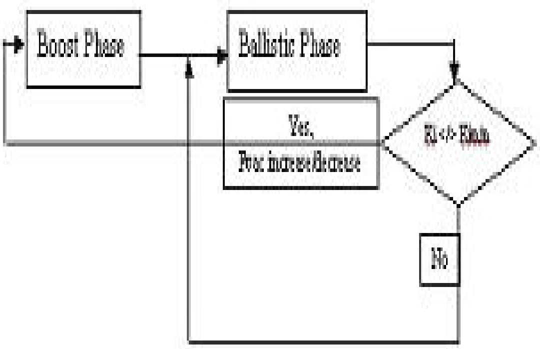

Fig 9 Flowchart for using optimal conditions

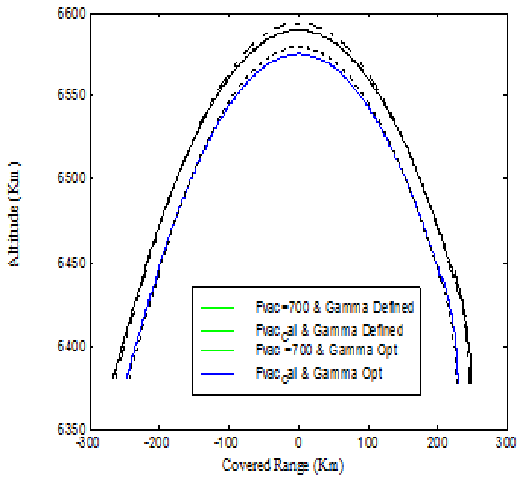

Fig 10 Trajectory Optimization Comparison

From Fig (9), we can note that the effect of Ki optimal increase the range with min 20 Km but the using of the flight path angle change the trajectory only but not change the range that covered but give us another benefit, is that the distance covered in the re-entry phase is optimal in heat sense and the angle of re-entry is optimized due to the discussion in section 5.2 and Fig 5.

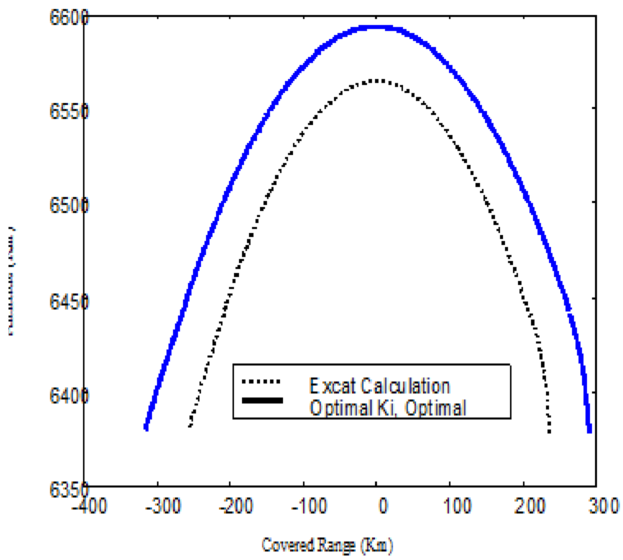

Fig 11 Effects of using optimal conditions

Covered Range (Km)

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Thermal Calculation on Re-entry point

Thermal protection is a crtical here in those cases which is used to be a metallic heat shield, which is implemented by a heat sink for the short heating waves. and well noted that delivery accuracy improved by increasing the values of b using small blunted slender shapes with increasing the impact velocity.

So the winds cannot affect the final trajectory. Thermal protection was provided by allowing the material at the surface of the heat shield to melt or vaporize thus transferring much of the heat back into the atmosphere. This method of thermal protection is referred to as "ablation," and the material that is applied to the vehicle's outer surface is called an "ablator."

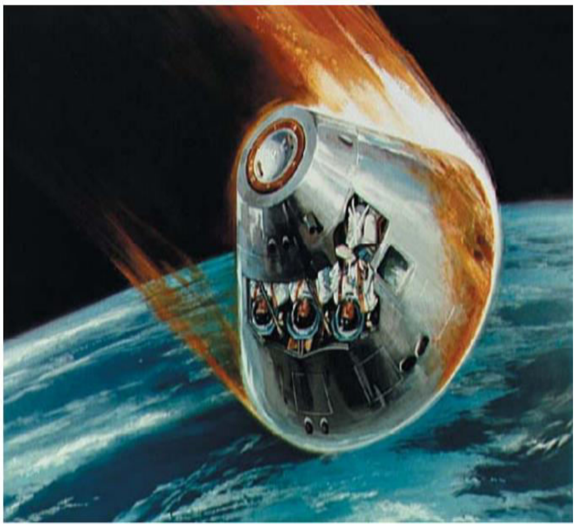

Fig 12 Apollo Capsule Re-entry shows that air friction causes the capsule to glow red hot.

Conclusion

This method is a good enough as a first step to find the optimal Ballistic missile trajectory. In addition, due to a lot of modules and its difficulties, this method can be more reliable if each module studied in details separately. And this will be worked on the next researches

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