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# DISCUSSING GENERATING FUNCTIONS AND DIFFERENTIAL EQUATIONS BY DIFFERENT METHODS

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**ABSTRACT**
Generating functions are presented here in their general form for the first time. Use of the generating function allows us to derive the terms of various polynomials. Moreover, we determine the nth term, an n, of the polynomial. When solving issues in physics, engineering, statistics, and operations research, special functions—and specifically hypergeometric functions and polynomials in one or more variables—are commonly needed. Several writers have recently paid attention to generation functions, summations, and transformations formula in the theory of special functions. The study of special functions relies heavily on generating functions, finite sum characteristics, and transformations. In light of the increasing relevance of generating functions, this dissertation includes specific classes of generating functions, including linear, bilinear, bilateral, double, and multiple generating functions for selected special functions and polynomials in one, two, or more variables. You can get these generating functions by utilising the series rearrangement method, integral operator approaches, Nishimoto's fractional calculus, or group theoretic methods.

**Keywords: -** Special Function, Equation, Generating function, Sciences, Application.

## I. INTRODUCTION

It is common knowledge that generating functions play significant roles in the investigation of a broad variety of potentially useful properties and qualities of the sequences that they create. These functions are responsible for producing. The generating functions are responsible for the creation of these sequences. To convert differential equations describing discrete-time signals and systems into algebraic equations, generating functions are another tool that may be employed. Another use for creating functions is seen here.

This makes it possible to simplify discrete-time system analysis as well as a wide variety of other problems that call for sequential fractional-order difference operators, operations research, and other areas of applied sciences. These advantages could also be applicable to a variety of other fields of applied research (including, for example, queuing theory and related stochastic processes).

## II. GENERATING FUNCTIONS

In the next section of this essay, we are going to do our best to explain the overall shape of the generating function. In addition, we have the ability to extract the polynomial functions as well as their coefficients, both of which are essential components in the development of special functions.

$$G(x,t) = \sum_{n=0}^{\infty} F_n(x)t^n, (8)$$

In order to derive the Fn (x) using the equation, we get here

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$$F_n(x) = \frac{1}{n!}\frac{\partial^n G(x,t)}{\partial t^n} \mid t = 0 \ (10)$$

Where the equation stands for the underlying framework of the method for aggregating the various polynomials as a whole. The next thing that has to be done is to find the coefficients of an, which are important in the special function. This may be found by following the instructions in the previous sentence. This is the next step that must be taken after the previous one. The revised version of the polynomial Fn(x) is presented below for your reference. It looks like this now:

$$F_n(x) = \sum_{n=0}^{\infty} a_n x^n \ (11)$$

Also we have

$$F_n(x) = \sum_{n=0} \frac{x^n}{n!}\frac{\partial^n F_n(x)}{\partial x^n} \ (12)$$

In conclusion, the equation gives us the opportunity to derive the following expression:

$$a_n = \frac{1}{n!^2}\frac{\partial^{2n} G_n}{\partial x^n \partial t^n}(x=0, t=0) = \frac{1}{n!}\frac{\partial^n F_n(x)}{\partial x^n}_{(x=0)}$$

## III. SOME POLYNOMIALS DEFINED BY GENERATING FUNCTIONS AND DIFFERENTIAL EQUATIONS

Research on the characteristics of polynomials is necessary, and the creation of new polynomials via the use of unconventional generating functions is not only intriguing but also necessary. The first person to characterise polynomials by making use of the generating function was Humbert (1921) $\Pi_{n,m}^{v}(x), n = 0,1,2,\ldots,$ might be any value from.

$$(1 - mtx + t^m)^{-v} = \sum_{n=0}^{\infty} \Pi_{n,m}^{v}(x) t^n. \ (14)$$

The generalisation of the Humbert polynomial of degree n was supplied by Gould (1965), $\Pi_{n,m}^{v}(x)$ who was also responsible for naming the polynomial. In the study that Milovanovi and Djordjevic carried out in 1987, difference operators were used, and the findings of the study ultimately led to the development of a differential equation for the function. $\Pi_{n,m}^{v}(X)$ By using the generating function, Lahiri (1971) provided a definition for the generalized Hermite polynomials as $H_{n,m,v}(x), n = 0,1,2,\ldots,$ where n may range anywhere from.

$$\exp(vtx - t^m) = \sum_{n=0}^{\infty} H_{n,m,v}(x)\frac{t^n}{n!} \ (15)$$

Gould and Hopper are responsible for the presentation of the other extension of Hermite polynomials by the generating function (1962).

$$x^{-a}(x-t)^a \exp\left(p(x^r - (x-t)^r)\right)$$

The scenario that is being analysed, in which a=0 has been set, is the same as the one stated by Bell (1934). We had a discussion regarding the possibility of developing Polynomials in Suzuki $Q_n(x; k, v), n = 0,1,2,\ldots,$ by making use of the following generating function, which has qualities that are comparable to those of the Humbert polynomials.

$$(1 - 2tx + t^k)^{-v} = \sum_{n=0}^{\infty} Q_n(x; k, v) t^n,$$

Where $k$ is an integer such that K>2 and V is a positive real number. Note that

$$\Pi_{n.m}^{v}(x) = Q_n(mx/2; m, v)$$

The polynomial $Q_n(x; k, v)$ is not an altogether novel concept either. Nonetheless, we provided a differential equation as an explanation for the function

$Q_n(x;k,v)$ Instead, this is stated out as an explicit expression, and it does not include any difference operators. By making x equal to zero and doing some more calculations, we were able to get the general solution to the differential equation. $Q_n(x;k,v)$ Satisfying that condition. This allowed us to do so. Dobashi (2014) is the title of the research paper in which we address the potential of establishing an extension of the Hermite polynomials by using the generating function. This study was just released.

$$\exp\left(t^k x - t^{k+j}\right) = \sum_{n=0}^{\infty} R_n(x;k,j)t^n, (16)$$

Taking it for granted that both k and j are positive integers. And the results that we obtained were similar to those that were found in the instance of $Q_n(x;k,v)$ In this particular instance, the general solution that corresponds to it is written down as a linear combination of functions that are stated by the application of ${}^2F_{k+j-1}$type hypergeometric functions. And the results that we obtained were comparable to those that were found in the instance of $Q_n(x;k,v)$The differential equations for $R_n(x;k,j)$and

R n (x;k,j) are the subject of the study for this work, as well as obtaining the general solutions for those differential equations when x=0 is taken into consideration. The debate of $Q_n(x;k,v)$can be found in, and the discussion regarding $R_n(x;k,j)$can be found in.

## IV. SOME POLYNOMIALS AND SPECIAL FUNCTIONS BY USING LIE LAPLACE TRANSFORMATION

When Courant and Hilbert were doing research in 1953 into the application of ordinary differential equations in the sphere of physics, they came into the concept of special functions by complete accident. As a direct consequence of this, the study of special functions eventually emerged as a distinct academic discipline. In addition, at the same time, Morse and Feshbach were looking at the ways in which special functions might be used to the research of difficulties that are related to the physical sciences. As a direct result of this, there have been developments made in the area of special functions as a consequence. On the basis of what he has seen, Paul Turan asserts that the history of special functions goes back a very, very, very long distance. This assertion is based on his observations. Euler, Legendre, Laplace, Gauss, Kummer, Riemann, and Ramanujan are just a few of the well-known mathematicians who were active in the 18th and 19th centuries and made significant contributions to the theory of special functions. Riemann and Kummer also played important roles in the development of the theory of special functions. The unique functions were the focus of study in the past, and for the same reasons they are the focus of research in the present day.

These include their application to a range of different subfields within physics and mathematics as well as their interaction with other subfields, such as number theory, combinatory, computer algebra, and representation theory. Other examples are also included. If a reader is interested in the subject matter, they should study the exceptional books that have been written on the subject by various authors Andrews, Rao, Rose, and Rainville. Miller's work helped expand Weisner's theory by establishing a connection between that theory and the factorization technique developed by Schrodinger. This was Miller's contribution to the scientific community. Miller's contribution to the development of Weisner's theory consisted of the following. He then went on to demonstrate a link between the theory and the work done by Infield and Hull, which broadened the applicability of the theory even more. Kalnins, Onacha, and Miller have all contributed to study on the topic of Lie algebraic characterizations of two-variable Horn functions. This line of inquiry has been pursued by all three researchers. In order to do this, they stretch a two-variable Horn function into a set of hyper geometric functions with a single variable. Because of this, a technique for the development of generating functions is eventually conceived of and developed. The discussion of hyper geometric functions in one, two, and even more variables takes up the majority of the space allotted to this section of the thesis. Following a discussion of the definitions of basic functions like the Gamma and Beta functions as well as the significant elements of these functions, these functions are then presented as prospective solutions to the issue that is now being considered.

**Laplace transformation**

The Laplace transform is an integral transform that is used often in the fields of physics and engineering. Additionally, it is capable of being utilised in a wide number of contexts due to the breadth of its applicability. Lf is the notation that is utilised in order to denote it (t). In the event that f(t) is a real-valued function that is defined for t values that are greater than 0, and in the case that both of these requirements are satisfied, then we may say that the given situation is true.

## V. CONCLUSION

Many novel functions have been developed within the framework of differential equations theory. Specialized operations are what we refer to here. The theory of special functions is fundamental to the formalisation of mathematical physics and applied mathematics. First established by Euler, Gauss, Laplace, Bessel, Legendre, Jacobi, Hermit, Laguerre, and others, then expanded upon by Whittaker, Watson, Ramanujan, Appell, Ragab, Hardy, Lebdev,Erdelyi, Chaundy, Bailey, and many others, and finally, continually refined by new achievements and suggestions within the context of applied sciences. In the fields of physics, engineering, statistics, and operations research, special functions, in general, and hypergeometric functions and polynomials in one or more variables, in particular, come up rather often in a broad range of issues. During the course of the last few years, a number of writers will have paid some attention to various aspects of theory pertaining to special functions, including generation functions, summations, and transformations formula. When it comes to the study of special functions, generating functions, finite sum characteristics, and transformations are all very important aspects. In light of the increasing significance of generating functions, this synopsis will include specific classes of generating functions. These classes include linear, bilinear, bilateral, double, and multiple generating functions for specific special functions and polynomials with one, two, or multiple variables. The series rearrangement approach, integral operator methods, Nishimoto's fractional calculus, and the group-theoretic method are the techniques that are used to produce such generating functions. Additionally, several transformations, fractional derivative formulae, and finite sum characteristics for specific hypergeometric functions and polynomials are provided, and a variety of special instances are derived from these qualities.

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