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TITLE: A STUDY OF MATHEMATICAL SOFTWARE RELIABILITY MODEL

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A STUDY OF MATHEMATICAL SOFTWARE RELIABILITY MODEL

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ABSTRACT

Mathematical model is basically a symbolic representation involving mathematical concepts/symbols and terminologies. Mathematical models are extremely powerful because they enable predictions to be made regarding a system. Further, these predictions provide a road map for further experimentation. Numerical models are utilized not just as a part of characteristic sciences and building disciplines but also in social sciences. Additionally, numerous physicists, engineers, analysts, financial experts, operations research examiners use scientific models most widely to pick the best strategy and test with the different alternative decisions. One of the fields where mathematical modeling has been vastly applied is reliability. The subject has been traditionally attached to hardware systems. Yet, with the expanding utilization of PCs in present times programming dependability has turned into a control in its own. Thus, making software reliability an important discipline in today's era. The aim of this study is to the Software Reliability Engineering is an emerging discipline whose importance cannot be undermined. It is receiving unprecedented attention from researchers. In this thesis, it was endeavored to develop more practical resource allocation approach catering to different fault removal models under dynamic environment. The software reliability growth model based on Half Logistic order statistics distribution is framed. The software reliability growth models are framed based on Non-Homogeneous Poisson Process. Algorithms and MATLAB programs for these reliability growth models are developed. Modeling is a science which needs creativity linked with deep knowledge of the methods appearing in many fields like bio engineering, financial engineering, environment industry, information and communication technology, applied mathematics etc.

KEYWORDS: Mathematical, Software Reliability Model, Numerical models, MATLAB programs

INTRODUCTION

In this paper, Software Reliability Growth model and Software Reliability Growth Model based on Order statistics of exponential distribution are framed. Algorithms and MATLAB programs used to frame the SRGMs are proposed. Various sets of data are analyzed using the proposed SRGM.

GOEL – OKUMOTO OR EXPONENETIAL GROWTH MODEL

The Goel-Okumoto Model is a simple NHPP Model. Let the random variable X be the cumulative time between failures. The probability density function of the exponential distribution is

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$$f(x) = be^{-bx} \qquad \text{... (2.1.1)}$$

where $x \geq 0$, $b > 0$, b is the rate or inverse scale parameter.

Its corresponding cumulative distribution function is

$$F(x) = 1 - e^{-bx} \qquad \text{... (2.1.2)}$$

Parameter estimation

Method of Maximum Likelihood is used to estimate the parameter. The likelihood function is given by

$$l = \prod_{i=1}^{n} be^{-bx_i} \qquad \text{... (2.1.3)}$$

The log-likelihood function is

$$\log l = \log \prod_{i=1}^{n} be^{-bx_i} \qquad \text{... (2.1.4)}$$

$$\log l = \sum_{i=1}^{n} \log be^{-bx_i} \qquad \text{... (2.1.5)}$$

$$= \sum_{i=1}^{n} \log b + \sum_{i=1}^{n} -bx_i \qquad \text{... (2.1.6)}$$

$$\log l = n \log b - b \sum_{i=1}^{n} x_i \qquad \text{... (2.1.7)}$$

Partially differentiating (2.1.7) with respect to b, and equating it to zero,

$$\frac{\partial}{\partial b} \log l = \frac{n}{b} - \sum_{i=1}^{n} x_i \qquad \text{... (2.1.8)}$$

$$\frac{\partial}{\partial b} \log l = 0 \qquad \text{... (2.1.9)}$$

$$b = \frac{n}{\sum_{i=1}^{n} x_i} \qquad \text{... (2.1.10)}$$

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NHPP Model of Goel – Okumoto SRGM

The mean value function m(x) and intensity function λ(x) for finite value exponential NHPP models are given, using (1.1.16) and (1.1.15), as follows

$$m(x) = a(1 - e^{-bx}) \qquad \text{... (2.1.11)}$$

$$\lambda(x) = abe^{-bx} \qquad \text{... (2.1.12)}$$

where, the expected number of failures, using (1.1.17), is given by

$$a = \frac{n}{1 - e^{-bx_n}} \qquad \text{... (2.1.13)}$$

GOEL-OKUMOTO OR EXPONENTIAL ORDER STATISTICS GROWTH MODEL

Let X1, X2, ..., Xn be random variables representing a sample of size n cumulative time between failures. Let X1:n, X2:n,................ Xn:n, be the original random variables so that X1:n ≤ X2:n ≤ ... ≤ Xn:n.

The probability density function of Goel-Okumoto (or) Exponential r th order statistics is given by

$$f_{r:n}(x) = r\binom{n}{r}b(e^{-bx})^{n-r+1}(1-e^{-bx})^{r-1} \qquad \text{... (2.2.1)}$$

where $x \geq 0$, $b > 0$, $1 \leq r \leq n$,

The cumulative distribution function is

$$F_{r:n}(x) = \sum_{i=r}^{n}\left[\binom{n}{i}(e^{-bx})^{n-i}(1-e^{-bx})^{i}\right] \qquad \text{... (2.2.2)}$$

Parameter estimation

Method of Maximum likelihood is used to estimate the parameter b. The likelihood function of Goel – Okumoto order statistics is

$$l = \prod_{i=1}^{n} r\binom{n}{r}b\left[e^{-bx_i}\right]^{n-r+1}\left[1-e^{-bx_i}\right]^{r-1} \qquad \text{... (2.2.3)}$$

The log – likelihood function is

$$\log l = \log\left[\prod_{i=1}^{n} r\binom{n}{r}b\left[e^{-bx_i}\right]^{n-r+1}\left[1-e^{-bx_i}\right]^{r-1}\right] \qquad \text{... (2.2.4)}$$

Using unconstrained optimization technique, the maximum of 'log l' is found.

NHPP model for Goel – Okumoto order statistics SRGM

The mean value function for this SRGM, from (1.1.16) and (2.2.2), is

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$$m(x) = a\left[\sum_{i=r}^{n}\binom{n}{i}(1-e^{-bx})^{i}(e^{-bx})^{n-i}\right] \qquad \dots(2.2.5)$$

The intensity function, from (1.1.15) and (2.2.1), is

$$\lambda(x) = abr\binom{n}{r}(e^{-bx})^{n-r+1}(1-e^{-bx})^{r-1} \qquad \dots(2.2.6)$$

Here, the expected number of failures, using (1.1.17) and (2.2.2), is given by

$$a = \frac{n}{\sum_{i=r}^{n}\binom{n}{i}(1-e^{-bx_n})^{i}(e^{-bx_n})^{n-i}} \qquad \dots(2.2.7)$$

ALGORITHM FOR GOEL-OKUMOTO SRGM

Step 1:Find the cumulative data of the time between failures

Step 2:Estimate b from (2.1.10)

Step 3:Using this b find the expected number of failures from (2.1.13)

Step 4:Find the control limits UCL, LCL and CL

Step 5:Estimate the mean value function in (2.1.11) at all failure numbers

Step 6:Then, find the successive differences of mean value functions

Step 7:Plot the mean value chart taking failure numbers along X-axis and successive differences along Y-axis.

Step 8:The failure numbers at which the mean value function is below LCL, detects the failure of the software.

MATLAB Program for Goel-Okumoto SRGM

```
u=[data];
n=length(u);
cum=u(1);sum1=u(1);
for i=2:n
  sum1=sum1+u(i);
  cum=[cum sum1];
end
x=cum;
```

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```
n=length(x);sum2=0;
for i=1:n
   sum2=sum2+x(i);
end
b=n/sum2;
a=n/(1-exp(-b*x(n)));
UCL=0.99865*a;
LCL=0.00135*a;
CL=0.5*a;
d=0;sum1=0;sd=0;
for i=1:n
   m=a*(1-exp(-b*x(i)));
   sum1=sum1+b*x(i);
   d=[d m];
    dummy=d(i+1)-d(i);sd=[sd dummy];
 end
 MLE=n*log(b)-sum1;k=1;
 AIC=-2*(MLE)+2*k+2*k*(k+1)/(n-k-1);
 BIC=-2*(MLE)+2*log(n);
 hold on
 plot(sd(:,3:n+1));
 lcl=LCL;flag1=lcl;ucl=UCL;flag2=UCL;cl=CL;flag3=cl;
 for i=1:n
```

```
    lcl=[lcl flag1];

    ucl=[ucl flag2];

    cl=[cl flag3];

end

plot(lcl);plot(ucl);plot(cl);
```

ALGORITHM FOR GOEL-OKUMOTO ORDER STATISTICS SRGM

Step 1:Find the cumulative data of time between failures.

Step 2:Choose the value of r

Step 3:Using minimization technique of non-linear unconstrained objective function, find the minimum of –log l in (2.2.4)

Step 4: $Max\ f(z) = -Min - f(z)$

Using this, find the maximum of log l multiplying the value of -log l by (-1). The values of b that gives the maximum of log l is the optimum value of b.

Step 5:Calculate the expected number of failures in (2.2.7) using b in step 4.

Step 6:Find the control limits UCL, LCL and CL.

Step 7:Estimate the mean value function in (2.2.5) at all failure numbers.

Step 8:Then find the successive differences of mean value functions.

Step 9:Plot the mean value chart taking failure numbers along X-axis and successive differences along Y-axis.

Step 10: The failure number at which mean value function is below LCL, detects the failure of the software.

CONCLUSION

Software Reliability Engineering is an emerging discipline whose importance cannot be undermined. It is receiving unprecedented attention from researchers. In this thesis, it was endeavored to develop more practical resource allocation approach catering to different fault removal models under dynamic environment. A systematic technique for the software project managers to allocate the resources for detection and correction, when a company has limited budget has been developed. It is observed that initially the path of detection effort increases but after a certain point of time, it attains a saturation level. Also, when most of the simple faults have been debugged, there will remain some hard and complex faults though detection of these faults will no longer remain difficult due to learning curve phenomenon. Due to this, the path of correction effort initially decreases and then attains a saturation point. GA is applied successfully to obtain optimum value of detection and correction effort when the company has limited budget. In addition to this, a trade-off between reliability and release time of a software project is examined. Using sensitivity analysis, we have observed the pattern of correction cost and variation in optimal allocation with respect to the parameters.

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