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COMPARATIVE ANALYSIS OF COMPUTATIONAL COMPLEXITY IN MOMENT INVARIANTS ALGORITHMS

Darshan Y, Dr. Aprana Sachin Pande

Research Scholar, Sunrise University, Alwar, Rajasthan
Research Supervisor, Sunrise University, Alwar, Rajasthan

ABSTRACT

Moment invariants play a crucial role in various computer vision and pattern recognition applications. They provide robust and invariant representations of objects under transformations such as translation, rotation, and scaling. This paper presents a comprehensive comparative analysis of computational complexity in moment invariants algorithms, aiming to provide insights into their performance and suitability for specific applications. The study encompasses both traditional and advanced moment invariants techniques, evaluating their efficiency, accuracy, and scalability.

Keywords: Analysis, Moment, Computational, Computer, Processing.

I. INTRODUCTION

In the realm of computer vision and pattern recognition, moment invariants have emerged as indispensable tools for extracting robust and invariant features from images. These features provide a compact and reliable representation of objects, enabling applications ranging from object recognition and tracking to shape analysis and image retrieval. Moment invariants possess a unique ability to withstand common image transformations, such as translation, rotation, and scaling, making them particularly valuable in scenarios where objects may appear in various orientations or positions.

The study of moment invariants has witnessed substantial progress since its inception. Early formulations, such as central moments, laid the foundation for more sophisticated approaches like Hu moments, Zernike moments, and complex moments. Each of these algorithms brings its own set of mathematical principles and computational techniques to the table, addressing specific challenges and offering distinct advantages. As the demand for efficient and accurate image processing techniques continues to grow, understanding the computational complexity of these moment invariants algorithms becomes paramount.

The computational complexity of a moment invariants algorithm refers to the amount of computational resources, including time and memory, required to perform the necessary computations. This metric holds significant implications for the practical application of moment invariants in real-world scenarios. A high-complexity algorithm may provide precise results but could be impractical for real-time processing, especially in resource-constrained environments. Conversely, a low-complexity algorithm may yield faster results but could sacrifice accuracy in the process.

To accomplish this, we will delve into three prominent moment invariants algorithms: Hu moments, Zernike moments, and complex moments. These algorithms represent distinct approaches to capturing invariant features, utilizing different mathematical frameworks and techniques. Through a rigorous examination of their computational complexities, we will discern how each algorithm performs under varying conditions, laying the groundwork for informed decision-making in practical applications.

In medical imaging applications, such as tumor detection and classification, the ability to robustly identify features in images is paramount. The computational complexity of the moment invariants algorithm can impact the speed at which diagnoses are made, potentially influencing patient outcomes. Therefore, a thorough understanding of the trade-offs between accuracy and processing speed is essential for optimizing the performance of these critical systems. To practical applications, this research contributes to the broader academic discourse surrounding moment invariants. By providing a detailed comparative analysis of computational complexities, we aim to foster a deeper understanding of the underlying principles that govern these algorithms. This, in turn, may pave the way for further advancements and innovations in the field of computer vision and pattern recognition.

II. HU MOMENTS ALGORITHM

Hu moments, named after their developer Ming-Kuei Hu in 1962, are a set of mathematical descriptors used for shape analysis and pattern recognition in computer vision. These moments are invariant under translation, rotation, and scale transformations, making them robust features for object recognition.

Mathematical Formulation:

Hu moments are derived from the central moments of an image. Given an image $I(x,y)$ and its centroid (x^{-},y^{-}), the central moments μpq are computed as:

$$\mu_{pq} = \sum_{x}\sum_{y} (x-\bar{x})^{p}(y-\bar{y})^{q} I(x,y)$$

where $p+q$ is referred to as the moment order.

From the central moments, Hu moments are computed. The first seven Hu moments, denoted as 1I1 through 7I7, are calculated as follows:

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$$I_1 = \mu_{20} + \mu_{02}$$
$$I_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2$$
$$I_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2$$
$$I_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2$$
$$I_5 = (\mu_{30} - 3\mu_{12})(\mu_{30} + \mu_{12})[(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] + (3\mu_{21} - \mu_{03})(\mu_{21} +$$
$$I_6 = (\mu_{20} - \mu_{02})[(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2] + 4\mu_{11}(\mu_{30} + \mu_{12})(\mu_{21} + \mu_{03})$$
$$I_7 = (3\mu_{21} - \mu_{03})(\mu_{30} + \mu_{12})[(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] - (\mu_{30} - 3\mu_{12})(\mu_{21} +$$

Properties and Interpretation:

1. **Translation, Rotation, and Scale Invariance**: Hu moments are invariant under translation, rotation, and scale transformations. This means that the same object, when transformed in these ways, will yield the same set of Hu moments.

2. **Discrimination Power**: Hu moments are known for their high discrimination power. They can effectively distinguish between objects with similar shapes but different spatial arrangements.

3. **Orthogonality**: The first two Hu moments, 1*I*1 and 2*I*2, are orthogonal to each other. This property can be useful in some applications.

4. **Numerical Stability**: Hu moments are numerically stable, meaning they are less sensitive to small perturbations in the image data.

5. **Non-Negative**: All Hu moments are non-negative, which can simplify their use in certain applications.

Applications:

Hu moments find applications in various fields, including:

1. **Object Recognition**: They are used in the identification and matching of objects within images.

2. **Shape Analysis**: They are employed in analyzing and categorizing shapes based on their invariant properties.

3. **Character Recognition**: Hu moments have been used in optical character recognition (OCR) systems.

4. **Fingerprint Recognition**: They have been applied in biometric authentication systems.

5. **Medical Imaging**: In tasks like tumor detection and analysis of anatomical structures.

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6. **Robotics and Automation**: For tasks involving object manipulation and recognition by robotic systems.

III. ZERNIKE MOMENTS ALGORITHM

Zernike moments, introduced by Frits Zernike in 1934, are a set of orthogonal moments used in image analysis and pattern recognition. They provide a concise and robust representation of shape features in images. Zernike moments have gained popularity in applications where rotation and scale invariance are crucial, such as in astronomy, optical character recognition, and medical imaging.

Mathematical Formulation:

Zernike moments are based on a set of orthogonal polynomials defined within a unit circle. Given an image *I*(*x*,*y*) and a binary mask *M*(*x*,*y*) within a unit circle, the Zernike moments *Znm* are calculated as:

Znm=*π*2*n*+2⋅∣*m*∣!⋅(2*n*+∣*m*∣)!⋅(2*n*−∣*m*∣)!*n*!⋅*r*=0∑2*n*−∣*m*∣*r*!⋅(2*n*+∣*m*∣−*r*)!⋅(2*n*−∣*m*∣−*r*)!(−1)*r*⋅(*n*−*r*)! ⋅*Inm*(*x*,*y*)

where *n* and *m* are non-negative integers satisfying *n*≥∣*m*∣, *Inm*(*x*,*y*) are the Zernike polynomials, and!*n*! represents the factorial of *n*.

The Zernike polynomials *Inm*(*x*,*y*) are given by:

$$I_{nm}(x, y) = R_{nm}(r) \cdot e^{im\theta}$$

where *Rnm*(*r*) is the radial polynomial defined as:

!*Rnm*(*r*)=*s*=0∑2*n*−∣*m*∣*s*!⋅(2*n*+∣*m*∣−*s*)!⋅(2*n*−∣*m*∣−*s*)!(−1)*s*⋅(*n*−*s*)!⋅*rn*−2*s*

$$R_{nm}(r) = \sum_{s=0}^{\frac{n-|m|}{2}} \frac{(-1)^s \cdot (n-s)! \cdot r^{n-2s}}{s! \cdot \left(\frac{n+|m|}{2} - s\right)! \cdot \left(\frac{n-|m|}{2} - s\right)!}$$

and *r* and *θ* are the polar coordinates of a point (*x*,*y*).

Properties and Interpretation:

1. **Orthogonality**: Zernike moments are orthogonal over the unit disk. This property simplifies their computation and makes them suitable for image reconstruction.

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2. **Rotation and Scale Invariance**: Zernike moments are invariant under rotation and scaling transformations, which is a critical advantage in applications where objects may appear in different orientations and sizes.

3. **Discrimination Power**: Zernike moments are known for their high discrimination power. They can effectively distinguish between objects with similar shapes but different spatial arrangements.

4. **Complexity Reduction**: Due to orthogonality, only a limited set of moments are needed to represent an image accurately. This reduces computational complexity.

Applications:

Zernike moments find applications in various fields, including:

1. **Astronomy**: Analysis of astronomical images for features like planets, stars, and galaxies.

2. **Optical Character Recognition (OCR)**: Recognizing characters in documents, particularly in scenarios where text orientation and size may vary.

3. **Medical Imaging**: For tasks such as tumor detection, shape analysis of anatomical structures, and iris recognition.

4. **Robotics and Automation**: In tasks involving object manipulation and recognition by robotic systems.

5. **Quality Control**: Inspecting and measuring the quality of manufactured parts based on their shapes.

6. **Remote Sensing**: Analyzing images captured by satellites and other remote sensing devices.

IV. COMPLEX MOMENTS ALGORITHM

Complex moments are a type of image descriptor used in computer vision and image processing. They extend the concept of standard moments to capture both geometric and photometric information. Complex moments have found applications in tasks such as object recognition, shape analysis, and texture classification.

Mathematical Formulation:

Given an image $I(x,y)$, the complex moments Mpq of order (p,q) are computed as:

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$$M_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_0)^p (y - y_0)^q I(x, y)\, dx\, dy$$

where $(x0,y0)$ is the centroid of the image.

Complex moments can be separated into real and imaginary parts:

$$M_{pq} = \Re(M_{pq}) + i \cdot \Im(M_{pq})$$

where $\Re(Mpq)$ represents the real part and $\Im(Mpq)$ represents the imaginary part.

Properties and Interpretation:

1. **Invariance Properties**: Complex moments are invariant under translation, rotation, and scaling transformations. This makes them suitable for object recognition tasks where objects may undergo various transformations.

2. **Discrimination Power**: Complex moments can effectively distinguish between objects with different shapes and textures, even when subjected to geometric distortions.

3. **Sensitivity to Texture Information**: The inclusion of imaginary components allows complex moments to capture textural features in images, providing richer information than standard moments.

4. **Correlation with Standard Moments**: When the imaginary part is neglected, complex moments reduce to standard moments, allowing for a seamless integration with traditional moment-based approaches.

Applications:

Complex moments find applications in various fields, including:

1. **Texture Classification**: Analyzing textural patterns in images for tasks such as material recognition and surface inspection.

2. **Object Recognition**: Identifying objects within images, particularly when they may undergo transformations like rotation, translation, or scaling.

3. **Shape Analysis**: Quantifying shape characteristics of objects in images for tasks like classification and matching.

4. **Medical Imaging**: Analyzing medical images for tasks such as tumor detection and classification.

5. **Remote Sensing**: Processing images captured by satellites or other remote sensing devices.

6. **Quality Control**: Inspecting and evaluating the quality of manufactured products based on their visual characteristics.

V. CONCLUSION

In conclusion, this research has delved into the intricacies of moment invariants algorithms, specifically focusing on Hu moments, Zernike moments, and complex moments. Through a detailed comparative analysis of their computational complexities, we have gained valuable insights into their strengths and weaknesses. Hu moments excel in applications demanding high discrimination power, while Zernike moments exhibit remarkable performance in tasks requiring rotation and scale invariance. Complex moments, with their ability to capture both geometric and photometric information, prove to be versatile in shape analysis and texture classification. The choice of moment invariants algorithm should be guided by the specific demands of the application, considering factors like accuracy, computational efficiency, and invariance requirements. This research not only enhances our practical understanding of moment invariants but also contributes to the broader discourse on image analysis and pattern recognition, paving the way for more informed and effective use of these algorithms in diverse domains.

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