International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

COPY RIGHT

IJIEMR Transactions, online available on 17th Sept 2021. Link :http://www.ijiemr.org/downloads.php?vol=Volume-10&issue=ISSUE-09

DOI: 10.48047/IJIEMR/V10/I09/20

Title **PAOR method for the solution of semi-nonlinear systems with linear diagonals**

Volume 10, Issue 09, Pages: 152-155

Paper Authors
Dr.V.B.Kumar Vatti, D.Kusuma, Dr.V.Kusuma Kumari, M.Santosh Kumar G.Chinna Rao

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

PAOR method for the solution of semi-nonlinear systems with linear diagonals

[1]Dr.V.B.Kumar Vatti [2] D.Kusuma [3]Dr.V.Kusuma Kumari [4]M.Santosh Kumar [5]G.Chinna Rao

[1]Professor, Dept.of Eng. Mathematics A U College of Engineering Andhra University Visakhapatnam, A.P, India
drvattivbk@yahoo.co.in

[2]Research Scholar/ Dept.of Engg Mathematics Andhra University Visakhapatnam, A.P, India
pkusuma1991@gmail.com

[3]Dean, Humanities and Basic Sciences Godavari Institute of Engineering and Technology Rajahmundry, A.P, India
dean.hbs@giet.ac.in

[4]Asst.Professor, Dept. of Basic Sciences and Humanities, Avanthi Institute of Engg & Technology, Narsipatnam, Visakhapatnam,
mksannthosh@gmail.com

[5]Asst.Professor, Dept. of Basic Sciences and Humanities, Avanthi Institute of Engg & Technology, Cherukupalli(v), Vizianagaram
golaganichinnarao@gmail.com

ABSTRACT

In this paper, we discuss parametric accelerated over relaxation (PAOR) method for the solution of semi-nonlinear systems with linear diagonals. A numerical example is considered to show the efficiency of this method.

Keywords:

Iterative methods, Jacobi, Gauss-Seidel, SOR,AOR, non-linear equations.

1 INTRODUCTION

We consider the semi nonlinear system with linear diagonals as introduced by V. B. Kumar Vattiet.al[5],is

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}f_{12}(x_2) + \ldots\ldots\ldots\ldots + a_{1n}f_{1n}(x_n) = b_1 \\ a_{21}f_{21}(x_1) + a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}f_{2n}(x_n) = b_2 \\ . \\ . \\ a_{n1}f_{n1}(x_1) + a_{n2}f_{n2}(x_2) + \ldots\ldots\ldots\ldots + a_{nn}x_n = b_n \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.1)$$

Let the matrix derived from(1.1) i.e.,

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{bmatrix} \text{............(1.2)}$$

be a positive definite matrix.

The PAOR method developed by V. B. Kumar Vattiet.al[6], for the solution of linear system

$$A\underset{\lrcorner}{X} = b$$

or

$$(D-L-U)\underset{\lrcorner}{X} = b \quad \text{.................. (1.3)}$$

where L,U, and D are strictly lower, strictly upper triangular parts and diagonal parts of the matrix 'A' respectively, is given by

$$[(1+\alpha)I - \omega\hat{L}]X^{n+1} = \{(1+\alpha-r)I + (r-\omega)\hat{L} + r\hat{U}\}X^n + r\hat{b} \text{............(1.4)}$$

$$(n = 0,1,2,.....)$$

where $\hat{L} = D^{-1}L$, $\hat{U} = D^{-1}U$, $\hat{b} = D^{-1}b$ ....(1.5)

The methods such as AOR, SOR, Gauss-Seidel and Jacobi can be realized form (1.4)

for the choice of (α, r, ω) = (0, r, ω), (0, ω, ω), (0,1,1), (0,1,0) respectively.

Let the minimum and maximum eigen values of the Jacobi matrix

$$J = \hat{L} + \hat{U} \quad \text{...........(1.6)}$$

in magnitude be $\underline{\mu}$ and $\overline{\mu}$ respectively.

And also, the choice of the parameters r and ω in the PAOR method given in [6] as

$$\omega = \frac{2(1+\alpha)}{1+\sqrt{1-(\overline{\mu})^2}}, \quad r = \frac{(1+\alpha)}{\sqrt{1-(\overline{\mu})^2}} \text{.............(1.7)}$$

if $\underline{\mu} = \overline{\mu}$ *and* $k = 1$

$$\omega = \frac{2(1+\alpha)}{1+\sqrt{1-(\overline{\mu})^2}}, \quad r = 1+\alpha+\omega+\frac{\overline{\mu}^2 - \underline{\mu}^2}{2} \text{.............(1.8)}$$

if $\underline{\mu} \neq \overline{\mu}$ *and* $k > 1$

$$\omega = \frac{2(1+\alpha)}{1+\sqrt{1-(\overline{\mu})^2}}, \quad r = \frac{\left[1+\alpha+\omega+\frac{\overline{\mu}^2 - \underline{\mu}^2}{2}\right]}{2} \text{............(1.9)}$$

if $\underline{\mu} \neq \overline{\mu}$ *and* $k < 1$

where

$$k = \frac{4(1+\alpha)(1-\sqrt{1-\overline{\mu}^2}) + (\overline{\mu}^2 - \underline{\mu}^2)\overline{\mu}^2 + 2(1+\alpha)\overline{\mu}^2}{4(1+\alpha) + (\overline{\mu}^2 - \underline{\mu}^2)(1+\sqrt{1-\overline{\mu}^2})} \text{ .... (1.10)}$$

We discuss Parametric accelerated over relaxation (PAOR) method in section-2 and a numerical example to show the effectiveness of this methods in the concluding section.

2 PARAMETRIC ACCELERATED OVER RELAXATION (PAOR) METHOD FOR THE SOLUTION OF (1.1)

Re-Writing the system (1.1) as done in [5], as

$$\left.\begin{array}{l} x_1 + \frac{a_{12}}{a_{11}} f_{12}(x_2) + ...+ \frac{a_{1n}}{a_{11}} f_{1n}(x_n) = \frac{b_1}{a_{11}} \\ \frac{a_{21}}{a_{22}} f_{21}(x_1) + x_2 + ...+ \frac{a_{2n}}{a_{22}} f_{2n}(x_n) = \frac{b_2}{a_{11}} \\ . \\ . \\ \frac{a_{n1}}{a_{nn}} f_{n1}(x_1) + \frac{a_{n2}}{a_{nn}} f_{n2}(x_2) + ...+ x_n = \frac{b_n}{a_{nn}} \end{array}\right\} \text{........................(2.1)}$$

Forming the matrix A_s by collecting the coefficients of functions as well as variables from (2.1), we have

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

$$A_s = \begin{bmatrix} 1 & \frac{a_{12}}{a_{11}} & . & . & . & \frac{a_{1n}}{a_{11}} \\ \frac{a_{21}}{a_{22}} & 1 & . & . & . & \frac{a_{2n}}{a_{22}} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \frac{a_{n1}}{a_{nn}} & \frac{a_{n2}}{a_{nn}} & . & . & . & 1 \end{bmatrix} \text{...........(2.2)}$$

The PAOR method for the solution of (2.1) is given by

$$x_1^{(n+1)} = [(1+\alpha-r)x_1^{(n)} - r\frac{a_{12}}{a_{11}}\frac{f_{12}x_2}{x_2}x_2^{(n)} - \text{..........} - r\frac{a_{1n}}{a_{11}}\frac{f_{1n}x_n}{x_n}x_n^{(n)} + r\hat{b}_1]/(1+\alpha)$$

$$x_2^{(n+1)} = [-\frac{\omega a_{21}}{a_{22}}\frac{f_{21}x_1}{x_1}x_1^{(n+1)} - \frac{(r-\omega)a_{22}}{a_{22}}\frac{f_{21}x_1}{x_1}x_1^{(n)} + (1+\alpha-r)x_2^{(n)} - \text{..........} - r\frac{a_{2n}}{a_{22}}\frac{f_{2n}x_n}{x_n}x_n^{(n)} + r\hat{b}_2]/(1+\alpha)$$

.

.

$$x_n^{(n+1)} = [-\frac{\omega a_{n1}}{a_{nn}}\frac{f_{n1}x_1}{x_1}x_1^{(n+1)} - \frac{\omega a_{n2}}{a_{nn}}\frac{f_{n2}x_2}{x_2}x_2^{(n+1)} - \frac{(r-\omega)a_{n1}}{a_{nn}}\frac{f_{n1}x_1}{x_1}x_1^{(n)} - \frac{(r-\omega)a_{n2}}{a_{nn}}\frac{f_{n2}x_2}{x_2}x_2^{(n)} - \text{.....} - (1+\alpha-r)x_n^{(n)} + r\hat{b}_2]/(1+\alpha)$$

$$\text{(n = 0,1,2,..........)} \quad \text{.....(2.3)}$$

The above method (2.3) in matrix notation can be expressed as

$$\begin{bmatrix} 1+\alpha & 0 & . & . & . & 0 \\ \omega\frac{a_{21}}{a_{22}}\frac{f_{21}(x_1)}{x_1} & 1+\alpha & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \omega\frac{a_{n1}}{a_{nn}}\frac{f_{n1}(x_1)}{x_1} & \omega\frac{a_{n2}}{a_{nn}}\frac{f_{n2}(x_2)}{x_2} & . & . & . & 1+\alpha \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \\ \\ x_n \end{pmatrix}^{(n+1)} =$$

$$\begin{bmatrix} 1+\alpha-r & -r\frac{a_{12}}{a_{11}}\frac{f_{12}(x_2)}{x_2} & . & . & . & -r\frac{a_{1n}}{a_{11}}\frac{f_{1n}(x_n)}{x_n} \\ -\frac{(r-\omega)a_{21}}{a_{22}}\frac{f_{21}(x_1)}{x_1} & 1+\alpha-r & . & . & . & -r\frac{a_{2n}}{a_{22}}\frac{f_{2n}(x_n)}{x_n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ -\frac{(r-\omega)a_{n1}}{a_{nn}}\frac{f_{n1}(x_1)}{x_1} & -\frac{(r-\omega)a_{n2}}{a_{nn}}\frac{f_{n2}(x_2)}{x_2} & . & . & . & 1+\alpha-r \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}^{(n)} + r\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \\ . \\ . \\ . \\ \hat{b}_n \end{pmatrix}$$

The PAOR iterative matrix is

$$P_s = ((1+\alpha)I - \omega\hat{L})^{-1}\left\{(1+\alpha-r)I + (r-\omega)\hat{L} + r\hat{U}\right\} \text{........ (2.4)}$$

The PAOR method converges if the spectral radius

P_s is less than one i.e.,

$$\rho(P_s) < 1 \text{ .... (2.5)}$$

3 NUMERICAL EXAMPLES

Example 3.1:

We consider a semi non-linear system with linear diagonals considered in V. B. Kumar Vatti et.al [5]

$$\left.\begin{aligned} 20x_1 - x_2^3 - x_3^2 &= 18 \\ -x_1^3 + 7x_2 - 2x_3 &= 4 \\ -x_1^2 - 2x_2^2 + 10x_3 &= 7 \end{aligned}\right\} \text{...........(3.1)}$$

whose exact solution is a unit vector.

The matrix A_S for the system (3.1) as obtained in (1.2) i.e.,

$$A_s = \begin{bmatrix} 20 & -1 & -1 \\ -1 & 7 & -2 \\ -1 & -2 & 10 \end{bmatrix} \text{....... (3.2)}$$

is positive definite whose Jacobi matrix J_S is

$$J_s = \begin{bmatrix} 0 & 1/20 & 1/20 \\ 1/7 & 0 & 2/7 \\ 1/10 & 2/10 & 0 \end{bmatrix} \text{........ (3.3)}$$

The eigen values of the Jacobi matrix are found to be 0.281822,0.042332 and 0.239490. And, hence $\underline{\mu} = 0.042332$ & $\overline{\mu} = 0.281822$. The relaxation parameter ω of SOR method as defined in (1.9) with α=0, is obtained as

$$\omega = 1.02068588 \text{ ....(3.4)}$$

The methods discussed in this paper are applied to obtain the solution of (3.1) up to an error less than 0.5×10^{-12} taking a null vector as an initial guess and

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

the results obtained are tabulated below along with the error $E=\sqrt{\sum_{i=1}^{n}\left|1-x_i\right|}$.

Table:1

Methods	Choices of (α,r,ω)	No. Of iterations took for the convergence (n)	Error (E)
Jacobi	(0,1,0)	40	$0.10523533e^{-5}$
Gauss-Seidel	(0,1,1)	22	$0.33848931e^{-6}$
SOR	(0,1.02068588,1.02068588)	20	$0.34962202e^{-6}$
AOR	(0,1.02975085,1.02068588)	19	$0.3994287e^{-1}$
PAOR	(-0.663,0.35989348,0.34397114)	14	$0.89382522e^{-1}$

4 CONCLUSION

It can be seen from the above tabulated results that the converging rate of PAOR method is superior to the methods discussed in this paper though the total error E is different in each of these methods.

5 REFERENCES

1. Kuo.M. Solution of nonlinear equations. Computers, IEEE Transactions on,C-17(9):897-898, Sep.1968.

2. Porsching.T.A. Jacobi and gauss-seidel methods for nonlinear network problems. 6(3),1969.

3. Varga, R.S Matrix Iterative Analysis, Prentice Hall, Englewood Cliffs NJ, 1962.

4. Vatti.V.B.K.,Numerical analysis: Iterative Methods I.K.International Publishing House Pvt.Ltd. (2016).

5. V.B.KumarVatti' , D.kusuma , M. Santhosh Kumar, Iterative methods for the solution of semi-nonlinear systems with linear diagonals, International Journal for Research in Applied Science & Engineering Technology 2021.

6. Vatti.V.B.K., G.Chinna Rao and Srinesh S.Pai, Parametric Accelerated Over Relaxation (PAOR) Method, Advances in Intelligent Systems and Computing, Vol.979, pp.283-288,2020.

7. Young, D.M., Linear Solution of Large Linear Systems, Academic Press, New York and London, 1971.