International Journal for Innovative Engineering and Management Research

A Peer Revieved Open Access International Journal

www.ijiemr.org

IJIEMR Transactions, online available on 18th Feb 2022. Link :http://www.ijiemr.org/downloads.php?vol=Volume-12&issue=Issue 02

DOI: 10.48047/IJIEMR/V12/ISSUE 02/73

Title **The Generated AUNU [7 4]- The Kown Linear Code [7 4 3] Using Hamming Code Method (U|U+V)**

Volume 12, ISSUE 02, Pages: 460-463

Paper Authors

M.J.Subhakar

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

The Generated AUNU [7 4]- The Kown Linear Code [7 4 3] Using Hamming Code Method (U|U+V)

M.J.Subhakar
HOD of Mathematics, Noble College, Machilipatnam
mjsubhakar@gmail.com

Abstract
In this communication, we enumerate the construction of a [7 4 2]- linear code which is an extended code of the [6 4 1] code and is in one-one correspondence with the known [7 4 3] - Hamming code. Our construction is due to the Carley table for n=7of the generated points of was permutations of the (132) and (123)-avoiding patterns of the non-associative AUNU schemes. Next, [7 4 2] linear code so constructed is combined with the known Hamming [7 4 3] code using the (u|u+v)-construction to obtain a new hybrid and more practical single [14 8 3] error- correcting code.

Keywords: Cayley tables; AUNU scheme; Hamming codes; Standard generator matrix; Extended code ; Reduced Row Echelon form (RREF); [n k d] linear code; (u|u+v) construction; Parity check matrix

Introduction
Historically, Claude Shannon's paper titled "A Mathematical theory of Communication" in the early 1940s signified the beginning of coding theory and the first error-correcting code to arise was the presently known Hamming [7,4,3] code, discovered by Richard Hamming in the late 1940s [1]. As it is central, the main objective in coding theory is to devise methods of encoding and decoding so as to effect the total elimination or minimization of errors that may have occurred during transmission [2] due to disturbances in the channel. The special class of the (132) and (123) avoiding Patterns of AUNU permutations has found applications in various areas of applied Mathematics [3]. The authors had reported the application of the adjacency matrix of Eulerian graphs due to the (132) - avoiding patterns of AUNU numbers in the generation and analysis of some classes of linear and cyclic codes [4,5], respectively. The authors utilized the Carley tables for n=5 [6] to derive a standard form of the generator/parity check matrix for some code. In this article [7], we enumerate the construction of a [7 4 2] – linear Code from the Carley table for n=7 of the generated points of was permutations of the (132) and (123) avoiding patterns of the non-commutative AUNU schemes [8]. The [742] linear code is then shown to be an extended [9] code of the [6 4 1] code and is in oneone correspondence with the [7 4 3]- Hamming Code. Moreover, the [7 4 2]- linear code so generated is [10,11] then combined with the known Hamming [7 4 3] code using the (u|u+v) construction method to obtain a new and more Practical single error correcting code with dimensions [12] n=14, k=8 and d=3.

Some Basic Concepts
Generator matrix
A generator matrix G for a linear code C is a kXn matrix for which the rows are a basis for C. If G is a generator matrix for C, then $C = \{aG\ a \in Fk\}$. G is said to be in standard form(often called the Reduce Echelon form) if ( ) k $G = I\ X$ where Ik is the $k \times k$ identity matrix.

International Journal for Innovative Engineering and Management Research
A Peer Revieved Open Access International Journal
www.ijiemr.org

The (U U +V) construction

Two codes of the same length can be combined to form a third code twice the length in a way similar to the direct sum of the codes construction. This is achieved as follows; Let Ci be an [n,ki,di] code for i∈{1,2}, both over the same finite field Fq. The (u u + v) construction produces a [2n,k1+k2,min(2d1,d2)] linear code

$$C = \{(u, u+v) \mid u \in C_1, v \in C_2\} \quad (1)$$

Remark

If Ci is a linear code and has generator matrix Gi and parity check matrix Hi, then the new code C as defined in (1) above has generator and parity check matrices as,

$$G = \begin{bmatrix} G_1 & G_1 \\ 0 & G_2 \end{bmatrix} \text{ and } H = \begin{bmatrix} H_1 & 0 \\ -H_2 & H_2 \end{bmatrix} \text{ respectively.}$$

Since the minimum distance of the direct sum of two codes does not exceed the minimum distance of either of the codes, then it is of little use in applications and is primarily of theoretical interest. As such, we for the purpose of this research concentrate on the $(u\ u + v)$ construction.

Example 1

Consider the binary [8,4,4] binary code C with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix},$$

Then C can be produced by combining the [4,3,2] code C_1 and the [4,1,4] code C_2 with generator matrices $G_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ and G_2[1 1 1 1] respectively, using the $(u \mid u+v)$ construction.

Methodology

Cayley tables

We consider the Carley table below, which is constructed using
An(132) for n=7 [2]

We now convert the entries of the Carley table above to the binary system using Modulus 2 arithmetic. Table 1, thus becomes;
The above table is the matrix G below;

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Clearly, all possible linear combinations of the rows of G generates a linear Code say C of length n=7 and size M=16 with the following Code words;

0 0 0 0 0 0 0	0 0 1 1 1 1 0	1 0 0 1 0 1 1	1 1 0 0 1 1 0
0 1 0 0 0 0 1	0 1 0 1 1 0 1	1 0 0 0 1 1 1	1 1 0 1 0 1 0
0 0 1 0 0 1 0	0 1 1 0 0 1 1	1 0 1 1 0 0 1	1 1 1 0 1 0 0
0 0 0 1 1 0 0	0 1 1 1 1 1 1	1 0 1 0 1 0 1	1 1 1 1 0 0 0

Now, since G has five rows and generates a code with sixteen code words, we seek a generator matrix for G. Deleting the first row and last column of G, we obtain a Matrix say $G^{II} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$

Next, we apply the following series of row operations on G^{II}, we have:

Θ	1	2	3	4	5	6	7
1	1	3	5	7	2	4	6
2	1	4	7	3	6	2	5
3	1	5	2	6	3	7	4
4	1	6	4	2	7	5	3
5	1	7	6	5	4	3	2

Θ	1	2	3	4	5	6	7
1	1	1	1	1	0	0	0
2	1	0	1	1	0	0	1
3	1	1	0	0	1	1	0
4	1	0	0	0	1	1	1
5	1	1	0	1	0	1	0

Table 1: Carley table for n=7 showing generated points of was permutations of (132) and (123)-avoiding patterns of AUNU scheme under the action of Θ.

1. $R_2 = R_1 + R_2$
2. $2\text{-}R_3 = R_1 + R_3$
3. $R_2 = R_3 + R_2$
4. $R_4 = R_1 + R_4$
5. $R_4 = R_4 + R_2$
6. $R_4 = R_3 + R_4$
7. $R_1 = R_1 + R_3$
8. $R_3 = R_4 + R_3$

Respectively as follows

$$G^{II}=\begin{bmatrix}1&0&1&1&0&0\\1&1&0&0&1&1\\1&0&0&0&1&1\\1&1&0&1&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&1&1&1&1\\1&0&0&0&1&1\\1&1&0&1&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&1&1&1&1\\0&0&1&1&1&1\\1&1&0&1&0&1\end{bmatrix}$$

$$\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\1&1&0&1&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&1&1&0&0&1\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&1&0&0&1\end{bmatrix}$$

$$\rightarrow\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&0&1&1&0\end{bmatrix}\rightarrow\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&0&1&1&0\end{bmatrix}\rightarrow\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&0&0&1\\0&0&0&1&1&0\end{bmatrix}$$

$$=\left[\begin{array}{cccc|cc}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&0&0&1\\0&0&0&1&1&0\end{array}\right]=G^{I}$$

Observe that *GI* above is a generator matrix in standard form [1],. *I* ( ) *k k n k i e G I X* × − = , but GI which is a generator matrix for C has words (rows) of length n=6, while C has word length n=7. Note also that all words in C has even weight, therefore extending the rows of GI by adding one digit each, to make the number of nonzero coordinates in each row of GI even, we obtain;

$$G^{I}=\left[\begin{array}{cccccc|c}1&0&0&0&1&1&1\\0&1&0&0&0&0&1\\0&0&1&0&0&1&0\\0&0&0&1&1&0&0\end{array}\right]$$

Which is the required generator matrix for our [7 4 2]-linear code C.

The $(U\ U+V)$ construction

In what follows, we combined the [7 4 2]-linear code constructed [3] with the known Hamming [7 4 3] code to obtain a Hybrid single error correcting linear code.

Now, the [7 4 2]-linear code has the following parameters; n=7,

k=4, d=2 and the known [7 4 3]-Hamming code has parameters; n=7,k=4, d=3.

On combining the two codes using the $(u\ u+v)$ construction, we shall obtain a [2(n=7),(k1=4+k2=4), min(2(d1=2), (d2=3)] =[2(7), 4+4, min(2(2), 3)]

=[14, 8, 3] - Linear code

Firstly, the [7 4 2]- linear code has generator and parity check matrices G_1 and H_1 as follows;

$$G_1=\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&0&0&1\\0&0&1&0&0&1&0\\0&0&0&1&1&0&0\end{bmatrix}\ and\ H_1=\begin{bmatrix}1&0&0&1&1&0&0\\1&0&1&0&0&1&0\\1&1&0&0&0&0&1\end{bmatrix}.$$

While for the generalize Hamming binary [7 4 3]-linear code, its generator and parity check matrices G_1 and H_2 are;

$$G_2=\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&1&0&1\\0&0&1&0&1&1&0\\0&0&0&1&1&1&1\end{bmatrix}\ and\ H_2=\begin{bmatrix}0&1&1&1&1&0&0\\1&0&1&1&0&1&0\\1&1&0&1&0&0&1\end{bmatrix}$$

respectively.

Now, by the $(u\ u+v)$ construction, our new code which is as a result of the combination of the above codes shall be a [14 8 3] – linear code whose generator and parity check matrices G0 and H0 are;

$$G_O=\begin{bmatrix}G_1&G_1\\0&G_2\end{bmatrix}=\begin{bmatrix}1&0&0&0&1&1&1&1&0&0&0&1&1&1\\0&1&0&0&0&0&1&0&1&0&0&0&0&1\\0&0&1&0&0&1&0&0&0&1&0&0&1&0\\0&0&0&1&1&0&0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&1&0&0&0&0&1&1\\0&0&0&0&0&0&0&0&1&0&0&1&0&1\\0&0&0&0&0&0&0&0&0&1&0&1&1&0\\0&0&0&0&0&0&0&0&0&0&1&1&1&1\end{bmatrix}$$

and

$$H_O=\begin{bmatrix}H_1&0\\-H_2&H_2\end{bmatrix}=\begin{bmatrix}1&0&0&1&1&0&0&0&0&0&0&0&0&0\\1&0&1&0&0&1&0&0&0&0&0&0&0&0\\1&1&0&0&0&0&1&0&0&0&0&0&0&0\\1&0&0&0&0&1&1&0&1&1&1&1&0&0\\0&1&0&0&1&0&1&1&0&1&1&0&1&0\\0&0&1&0&1&1&0&1&1&0&1&0&0&1\end{bmatrix}$$

As such, a single error correcting code of length n=14, dimension k=8 and Hamming distance d=3 has been constructed.

Findings

The Carley table for n=7 of the generated points of the (132) and (123)-avoiding patterns of the non-associative AUNU schemes has been used to construct a [742]-linear code C which is an extended code of the [641] code. Moreover, the [742]-linear code so generated is then utilized using the (u|u+v) construction method to obtain a new and more practical single error correcting code with dimensions n=14, k=8 and d=3.

Conclusion

This paper has further pointed the applicability of the AUNU schemes in the direction of coding theory, i.e., Codes, generation and analysis.

References

1. Ibrahim AA, Chun PB, Mustafa A, Abubakar SI (2016) A Standard generator/ Parity–check Matrix for Codes from the Carley Tables due to the nonassociative (123)-avoiding Patterns of AUNU numbers. Journal of the Nigerian Association of Mathematical Physics 35.
2. Ibrahim AA, Abubakar SI (2016) Non-Associative Property of 123-Avoiding Class of Aunu Permutation Patterns. Advances in Pure Mathematics, 6: 51-57.
3. Huffman WC, Pless V (2003) Fundamentals of error correcting codes. Cambridge University Press pp:18.
4. Chun PB, Ibrahim AA, Garba AI (2016) Algebraic theoretic properties of the avoiding class of AUNU permutation patterns: Application in the generation and analysis of linear codes. International Organization for Scientific Research (IOSR). Journal of Mathematics 12: 1-3.
5. Chun PB, Ibrahim AA, Garba AI (2016) Algebraic Theoretic Properties of the Non-Associative Class of (132)-Avoiding Patterns of Aunu Permutations: Applications in the Generation and Analysis of A General Cyclic Code. Computer Science and Information Technology 4: 45-47.
6. David J, John-Lark K (2011) Selected unsolved problems in Coding theory .
7. Hoffman DG, Leonard DA, Lindner CC, Phelps KT, Rodger CA et al. (1992) Coding Theory: The Essentials. Mercel Dekker.
8. Willem HH (2011) Matrices for graphs, Designs and Codes. Coding theory and Related Combinatorics.
9. Ibrahim M, Ibrahim AA, Yakubu MA (2012) Algebraic theoretic properties of the (132)-Avoiding class of Aunu patterns application in Eulerian graphs. Journal of Science and Technology Research.
10. Vasantha WB, Florentin S, Ilanthenral K (2010) Supper special Codes Using Supper Matrices. Infolearnquest, Ann Arbor .
11. Usman A, Ibrahim AA (2011) A new generating function for AUNU patterns: Application in integer Group Modulo n. Nigerian Journal of Basic and Applied Sciences 19.
12. Bruck RH (1958) A survey of the binary system. Springer Berlin.